11027-6001-RP-00

FINAL REPORT
DEVELOPMENT OF DIGITAL COMPUTER PROGRAM FOR THERMAL NETWORK CORRECTION

PHASE I – INVESTIGATION/FEASIBILITY STUDY

JANUARY 1969

Prepared by

T. Ishimoto
H. M. Pan
J. D. Gaski
E. B. Stear

Prepared for
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
Manned Spacecraft Center
Houston, Texas

NASA Contract 9-8289

TRW
SYSTEMS GROUP

ONE SPACE PARK • REDONDO BEACH, CALIFORNIA 90278

DEVELOPMENT OF DIGITAL COMPUTER PROGRAM FOR THERMAL NETWORK CORRECTION

PHASE I - INVESTIGATION/FEASIBILITY STUDY

FINAL REPORT

T. Ishimoto, H. M. Pan, J. D. Gaski, E. B. Stear

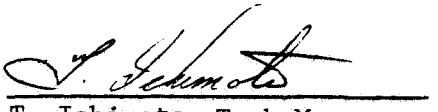
January 1969

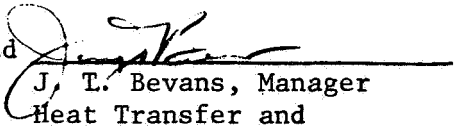
TRW Systems Group

Prepared for
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
Manned Spacecraft Center
Houston, Texas

NASA Contract 9-8289

Prepared for
National Aeronautics and Space Administration
Manned Spacecraft Center
Under Contract NASA 9-8289

Prepared 
T. Ishimoto, Task Manager

Approved 
J. T. Bevans, Manager
Heat Transfer and
Thermodynamics Department

ACKNOWLEDGEMENT

A number of persons have actively participated in this Phase I Study. Their efforts have materially aided whatever success that has been attained. Near the beginning of this study, a discussion with Dr. W. M. Lear, TRW Systems Group, on the Kalman filtering was of considerable help. The assistance of M. W. Alford on the use of Program MAFIA was very important in the present effort. The programming effort of L. C. Fink who generated the program on the least squares method with heat flux residue and assisted in the modification of CINDA-3G has been particularly valuable.

The secretarial services provided by Mrs. S. Higashi and Mrs. P. Varney is also gratefully acknowledged.

The authors are also particularly grateful to Mr. J. T. Bevans, Manager, Heat Transfer and Thermodynamics Department, for his encouragement during this Phase I Study and for his foresight to look into the general problem of parameter estimation prior to this study.

In addition, the authors express their appreciation to Mr. R. Dotts, NASA/MSC, who has monitored this program in a manner that has allowed the Phase I Study to yield maximum results.

TABLE OF CONTENTS

TABLE OF CONTENTS (continued)

TABLE OF CONTENTS (continued)

TABLE OF CONTENTS (continued)

TABLES

TABLES Continued

TABLES Continued

ILLUSTRATIONS

SUMMARY

The overall objective of this program is to develop an accurate digital computer program update and correction logic coupled with a spacecraft thermal analysis prediction capability; the program consists of three consecutive phases. Phase I is an investigation/feasibility study period whereby the feasibility of analytical error analysis as applied to thermal network solutions is established.

A literature search revealed two basic classes of correlation methods, one-pass and sequential. Within the framework of each class are numerous variations, each perhaps suited for specific requirements. A number of potentially suitable correlation methods selected from a list of techniques reported in literature were studied and screened, resulting in the selection of five methods for further study. These methods were: (1) Kalman filter; (2) Program MAFIA; (3) Quasilinearization; (4) Least Squares with net heat flux residual; and (5) Regression Analysis. The latter two were studied to obtain baseline information.

The results in many respects were found to be quite promising and in other respects, less so. The chief concern is the accommodation of the sparsity of temperature measurements. It is also clear that if a high percentage of temperature measurements are available or that the measurements are properly located, either the one-pass (Program MAFIA or Quasilinearization) or the sequential method (Kalman filter) can satisfactorily correct the parameters of a thermal network. However, the results were not of sufficient depth to realistically permit the choice of one method over the other. As a result both the one-pass method and the sequential method were recommended for further study.

NOMENCLATURE

$[A]$	= weighting matrix for the a priori estimates
$[A]$, P_o	= state error covariance matrix for the a priori estimates
$[B]$, B	= measurement weighting matrix (time varying gain)
C_i	= heat capacitance for node i
I	= identity matrix
$[J]$, J	= state error covariance matrix
$[M]$, M, H	= measurement matrix
Q_i	= heat input for node i
T_i	= temperature at node i
T_i^*	= random noise corrupted temperature measurement at node i
$[U]$, U	= state updating matrix
$[W]$	= weighting matrix for the random measurement noise
$[W]$, R	= measurement noise covariance matrix
a_{ij}	= linear conductor joining nodes i and j
b_{ij}	= radiation conductor joining nodes i and j
p	= probability function
$\underline{p}$	= parameter vector
t	= time
$\{x\}$, $\underline{x}$	= state vector
$\{y^*\}$, $\underline{y}^*$	= random noise corrupted measurement vector
$[\phi]$	= state transition matrix
η, $\underline{W}$, $\{W\}$	= random noise vector
Σ	= summation
Π	= product
$\|\| \phi \|\|_W$	= $\phi^T W \phi$

NOMENCLATURE (continued)

| | = absolute value

E( ) = expected value

f( ), g( ) = function of

1.0 INTRODUCTION

The program entitled "Development of Digital Computer Program for Thermal Network Correction" consists of three consecutive phases: (1) Phase I, Investigation/Feasibility Study; (2) Phase II, Program Development; and (3) Phase III, Demonstration and Application. Each of these phases is of 6-months duration. Phase I is an investigation/feasibility study period whereby the feasibility of analytical error analysis as applied to thermal network solutions is established. Phase II will be a programming effort to refine and code the NASA/MSC approved error analysis approach(s). Phase III will an effort to evaluate and demonstrate the capabilities of the computer program and sub-routines coded in Phase II.

The overall objective of this program is to develop an accurate computer program that updates and corrects a thermal network and one that is coupled with a spacecraft thermal analysis prediction capability. The analytical methods are to be developed with the ultimate intent to use test/post-flight data for correlation with analytical results.

Achievement of the objectives was considered to be best pursued by the multi-step process as indicated by the three consecutive phases. The Phase I effort was also segmented with literature search followed by a number of screening procedures to arrive at a select group of correlation methods that were studied in more detail. Related tasks included programming the thermal model error assessment considerations.

In the sections to follow are the results of the Phase I study which is presented sequentially in the multi-step fashion.

2.0 REQUIREMENTS AND LITERATURE SURVEY

2.1 REQUIREMENTS

A literature search to be effective must accommodate the basic objectives as well as the numerous constraints that are imposed. In order to place the literature survey in a proper perspective, the overall objectives of the present program and the basic constraints to be followed are discussed.

2.1.1 Objective

The overall objective of the program is to develop an accurate digital computer program update and correction logic coupled with a spacecraft thermal analysis prediction capability. The program to be developed is to be specifically directed towards development of analytical methods for computerized error and analysis and updating of the thermal networks with test/post-flight data results used for correlation of analytical simulation results.

2.1.2 Constraints

The basic constraints not only apply to the correlation methods directly but from practical considerations must also apply to system programming aspects and theoretical model characteristics. The more important considerations are discussed below:

2.1.2.1 Experimental Data

Experimental data are necessary to evaluate the correlation methods that are to be studied in detail. However, the Phase I computer results are to be used in lieu of experimental data. Since the numerical results are near-perfect, subject to only numerical errors, the correlation method(s) that is (are) ultimately selected for Phase II must also accommodate the inaccuracies of experimental data that are to be provided by NASA/MSC.

At the beginning of Phase I, the characteristics of the experimental data were not defined since since computer results were to be used during this phase, but it was recognized that the experimental data must be defined before the correction methods can be evaluated.

At present, it is assumed that experimental data will be smoothed by NASA/MSC and a tolerance band will be specified. The nature of the tolerance band has not been specified but it is assumed that it will be a random noise.

Another aspect of the experimental data is sparsity. It is recognized that temperature data will not be available at every nodal location used in the theoretical model but the extent of the sparsity depends upon the type of test. For example, an engineering model test will have many more measurements than say a flight article. Thus the correlation method to be a practical tool must account for the sparsity that exists on all thermal tests. It must also be recognized that the larger the sparsity the less accurate will be the correlation methods.

2.1.2.2 Systems Programming

A correlation method suitable for a small model of limited number of parameters may not be suitable for a large model of many parameters. The objectives of the present study present particular problems from even size considerations alone since the ultimate objective is to update a thermal model that may have as many as 1500 nodes; the number of parameters will be in the thousands. Processing of the data will be enormous, let alone updating the network. Thus the system programming aspects will be equally as important as the correlation method itself. Some of the system aspects would be the identification and bounding of the soft parameters, input of the test data, parameter correction subroutine, rerun and verification.

2.1.2.3 Theoretical Model

The types of errors that make-up the difference between measured and predicted temperature are mainly three in nature; these sources of errors are: (1) functional form of the model; (2) measurement errors; and (3) incorrect parameter values. If the functional form of the model is not sufficiently accurate, a new model must be constructed. Thus the parameter estimation methods will be applicable only if the errors are confined to measurement errors, biased and random, and parameter inaccuracies.

The physical system considered in the present study is one that can be reduced to a theoretical model which in turn can be represented by a thermal network of resistances and capacitances. The set of equations that describe the model can be expressed as follows:

$$C_i \frac{dT_i}{dt} = Q_i + \sum_{j=1}^{r} a_{ij}(T_j - T_i) + \sum_{j=1}^{r} \sigma b_{ij}(T_j^4 - T_i^4) \qquad (2\text{-}1)$$

$$i = 1,2,\ldots,n$$

$$T_j = \text{constant for } n<j\leq r$$

The parameters to be corrected will be found among the C_i's, the Q_i's, the a_{ij}'s, and b_{ij}'s. Simultaneously, an appreciable number of T_i's can be expected to be non-measured and thus considered to be unknown.

2.2 LITERATURE SEARCH

The correction of thermal model parameters from noisy temperature measurements belongs to the class of problems designated estimation problems; the term noisy is defined here to mean random errors. In general an estimation problem involves the identification and estimation of the parameters and states of a physical system from noisy measurements of some observable quantities of the system.

With the overall objectives and the basic constraints as general guidelines, a technical literature search encompassed a number of potential sources of information such as NASA STAR, Joint Automatic Control Conferences, and Computer Conferences. Not surprisingly, numerous articles in the general area of automatic control and statistics were found that offered pertinent information related to the problems of parameters correction. The literature search revealed papers in regression analysis [1-6]*, the method of least squares [7-17], the method of recursive least square [16-19], the method of maximum likelihood [1,6,17]; the method of Bayesian estimation [6,17,20], the method of invariant imbedding [21-23], the

* Superscript numbers refer to the references in the Bibliography Section

method of quasilinearization[24,25], the method of Kalman filtering [16,17,26-30], and sensitivity analysis which may be an integral part of any parameter correction scheme [31,32]. These methods can be categorized into two different classes. One class denoted as one-pass methods involves the estimation of parameters and states at some epoch (time), whereby all the measurements are processed in one pass; the methods in this class include regression analysis, least squares, maximum likelihood and quasilinearization. The other denoted as sequential methods involves the estimation of parameters and states sequentially, whereby the measurement data are processed sequentially and new estimates of the parameters are obtained after each set of measurements is processed. The methods of sequential estimation include Kalman filtering, invariant imbedding, and Bayesian estimation. These methods are discussed below but selected ones are presented in more detail in the next section.

2.2.1 One-Pass Methods

The method of least squares is a classical method that is widely used. In its simplest form, it involves the formation of a criterion function, ε, which is the sum of the squares of the difference between the computed model response and a set, y, of m experimental observations of a scaler output. The observations can be weighed if desired. The least squares estimate of the parameters, p, and states, x, are defined as the values of p and x that minimize ε. The method of least squares has been used in such areas as the determination of regression parameters in regression analysis, and the estimation of states and parameters for both linear and nonlinear dynamic systems.

To estimate the temperature, some scheme must be incorporated such that the heat balance equations are solved with the measured temperatures as boundary conditions; this then becomes a multi-point boundary value problem. To solve the multi-point boundary value problem, the method of quasilinearization can be applied [24,25]. The method of quasilinearization in essence in a method whereby the governing nonlinear heat balance differential equations are linearized by the 1st order Taylor series expansion about some nominal temperature

profiles and nominal parameter values. The nominal values are defined by the a priori estimate of the initial conditions. The linearized heat balance equations are solved assuming unknown parameter values and initial conditions. Improved temperature profiles are obtained as a function of these knowns. These unknowns are then determined in the least squares sense by minimizing the square of the difference between the improved temperature profiles and the measured temperatures. After all the data have been processed a new and better estimate of the unknowns is obtained. The results are then used to obtain new temperature profiles and the whole problem is rerun. This process continues (generally several times) until no further change in these unknowns are observed. This method also applies when the temperature is not measured at every nodal location.

Toussaint[12] applied the method of least squares for the correction of thermal model parameters. However, in lieu of using temperature as the measurement vector, Toussaint minimized the net heat flux residual at each nodal location over some specified time span. Unfortunately, this method will not accommodate the condition of sparse measurements. Toussaint concluded that when no measurement error is taken into account, a maximum of two parameters per node can be corrected.

Allison[13,14] applied the method of least squares to the problem of parameter estimation by using input-output measurements. This was done by forcing a mathematical model of the system with the measured input and adjusting model parameters to minimize the mean squared difference between the measured and model outputs. Allison shows that the classical sensitivity analysis method for minimizing the mean squared error is identical in principle to the method of quasilinearization.

The solution to a least squares estimate in general involves the solution of a set of n algebraic equations (the normal equations). When the number of parameters is small and the equations are well conditioned, one can use any reasonable scheme to obtain the solution on a digital computer. When the number is large and the equations are ill-conditioned, it is very difficult to arrive at a solution. Even if a solution is obtained, it may be difficult to ascertain the validity of the solution

in either a numerical or statistical sense. Many methods have been proposed to solve this problem. Rouch[7] proposed a numerical scheme whereby the parameters which make the problem ill-conditioned are eliminated as the set of normal equations is being solved by using the Gauss elimination scheme. Golub[8] proposed a method of matrix partition based on an orthogonal Householder transformations[33].

2.2.2 Sequential Methods

Kalman[26,28], Kalman and Bucy[27] were the first to propose a scheme whereby the states and parameters of a linear dynamic system can be estimated sequentially. This is the method of Kalman filtering which is a minimum variance estimate whereby the variances of the states and parameters to be estimated are minimized. The Kalman filter can also handle nonlinear systems. This is done by linearizing the nonlinear equations about some nominal trajectory (temperature history for the present study)[16,29]. Browne [30] has applied the Kalman filter to correct the parameters of a thermal model. In his study, he neglected the radiation coupling terms and thus eliminated the nonlinearity in the governing heat balance differential equations. This approximation allowed the direct application of the Kalman filter. Numerical results were not presented in his paper.

2.2.3 Comment on the One-Pass and Sequential Methods

The essential difference between the one-pass estimate and sequential estimate is that the former estimates the unknowns at some epoch by expressing the unknowns at any time point in terms of the unknowns at the particular chosen epoch. The latter estimates the unknowns at some time, t, and uses the new estimate at time, t, to project it forward to some new t+1; this projected estimate is used as the a priori estimate for the processing of data at time, t+1. This process continues until all until all data are processed.

The nature of the estimation problem governs the choice between the one-pass estimate and the sequential estimate. In general, the sequential estimation scheme is primarily used for on line estimation where the current estimate of the states and parameters are desired rather than the states and parameters at some epoch (time) is desired. In problems where large number of parameters are

involved (for example thermal nodel parameter estimations), it is sometimes preferable to use the sequential estimate since it only requires the inversion of an m x m matrix where m is the number of nodes in the in the model. The one-pass estimate necessitates the inversion of p x p matrix where p is the sum of the number of parameters and the number of nodes in the model, (p is often times very much greater than m).

Other sequential schemes have been developed for the estimation of states and parameters. These are the recursive least squares and invariant imbedding. The recursive least squares is a scheme whereby the states and parameters of a system can be estimated in a least squares sense without recalculating the entire least squares problem when additional data are available. Invariant imbedding is essentially an approximation method whereby the states and parameters of nonlinear dynamic systems can be estimated using the method of least squares.

The problem of states and parameters estimation can also be viewed from probability theory. The classical method is the maximum likelihood estimate[1,6,17] whereby the value of the parameters or variables to be estimated appearing in the probability distribution function is chosen in such a way as to maximize the likelihood function, and the likelihood function is the probability function when regarded as a function of the parameters. Ho[20] formulated the estimation problem from the Bayesian decision theoretic viewpoint. In this formulation, the a posteriori conditional probability distribution, $p(x/y)$, is derived, and the optimal estimate is obtained by evaluating the conditional mean, the median, or mode of the posteriori conditional probability depending on the criterion function used.

Although each of these methods appears to be distinct, these methods are indeed related. This relationship can be shown from the identification of the governing parameters. For example, the method of recursive least square can be identified with the method of Kalman filtering if proper assumption is made of the a priori estimate and that the weighting function on the measurement is identified as the inverse of the measurement noise covariant matrix. Upon proper manipulation of the governing equations, it

is also possible to convert a sequential estimator to a one-pass estimator or vice versa.

Another important difference between the sequential estimate and the one-pass estimate lies in the linearization of nonlinear system. For the one-pass estimate, the linearization is made about some nominal a priori estimate of the state at epoch (time). Upon the processing of all the data, a new and better estimate at this epoch (time) is obtained and used to provide the new a priori estimate, and the problem is rerun. This process continues until the change of the estimate as epoch (time) is within a specified allowance. In the sequential estimate, a new estimate of the state is obtained after each measurement is processed; this new estimate is used to provide a new nominal trajectory. As more and more measurements are processed, the estimated state and time value of the state come closer together and the linearization assumption is better and better satisfied[29]. However, if the equations governing the system is highly nonlinear and the linearized model is not correct, this will tend to drive the estimates slowly from the time state. This will in itself introduce errors due to nonlinearities[16].

Additional differences between the various methods studied involved the use of a priori knowledge and the use of statistical assumptions concerning the measurement noise and the states of the system. The various methods that require the use of some form of least squares formulation require no knowledge of the statistics concerning the states and the measurement noise and requires no a priori knowledge of the states and parameters. However, if a priori knowledge of the unknown states and parameters and measurement noise statistics are available, they can be incorporated in the calculation respectively as additional measurements and measurement weighting functions.

In applying these various methods to correct thermal model parameters, an ideal situation would be the availability of measured temperature histories for each of the nodes in the model. When measured transient temperature histories for every node are available, theoretically,

all the methods studied are applicable to the correction of thermal model parameters. Often in thermal model testing, only a limited number of nodes are measured, and it is necessary to correct the thermal model parameters using these limited temperature measurements. Theoretically all the methods studied with the exception of deterministic methods and regression analysis are capable of handling this situation. However, for the method of least squares, the criterion function to be used must be the square of the difference between the measured temperature and the computed temperature, not the net heat flux residual. The criterion function used by Toussaint[12] will not be able to handle this situation.

The method of maximum likelihood requires the knowledge of measurement noise statistics and no a priori knowledge of the parameters. If the measurement noise obey Gaussian statistics, it can be shown that this method is equivalent to the method of least squares provided the weighting function in the least squares formulation can be identified as the inverse of the measurement noise covariance matrix[1,17].

The method of Kalman filtering requires the assumption of a priori knowledge of the unknown states and parameter and their corresponding covariances; the measurement noise statistics is also required.

The method of Bayesian estimation requires the knowledge of a priori statistics concerning the unknown states and parameters and the measurement noise statistics. If the unknown states and the measurement noise obey Gaussian statistics, it can be shown that this method and the method of Kalman filtering are equivalent upon proper manipulation[17,20].

2.2.4 Sensitivity Analysis

Sensitivity analysis represents an integral part of any parameter estimation (correction) scheme since it can have a number of useful functions. Identification of hard and soft parameters and evaluation of temperature variance due to the uncertainties of the input information are two of the important functions. The term soft parameter is defined here to mean a parameter that has a larger uncertainty whereas a hard parameter is defined to mean

a parameter with a very small uncertainty. Some of the parameter correction scheme such as quasilinearization utilizes sensitivity analysis directly. Excellent sources of information on sensitivity analysis are references 31 and 32.

2.2.5 Summary

The literature search revealed two basic classes of correlation methods, one pass and sequential; these methods have been applied to small systems. Within the framework of each class are numerous variations, each perhaps suited for specific requirements. The requirements of the present program are unique from primarily size considerations, which is manifested by the large number of temperatures and parameters that must be considered.

Since both the one-pass and sequential methods have been successfully applied to small non-thermal systems, application of these methods to small thermal systems can also be expected to be successful with perhaps the single concern being parallel resistors. In a thermal system, conduction and radiation resistances in parallel are quite prevalent. Application of these methods to a large thermal system remains a major consideration.

In summary, the correlation methods revealed in literature are basically similar and can be expected to apply to small thermal systems. The choice between the one-pass and the sequential methods would perhaps be dictated from numerical solution and programming considerations. Variations within each class would merely affect the basic numerical and programming problems. In the section to follow, several of the more interesting and potentially suitable methods are presented. From this group of techniques, three were selected for a detailed examination by the use of the five- and twenty-node models.

3.0 METHODS OF PARAMETER ESTIMATION

The literature survey as presented and discussed in the previous section revealed two basic categories of correlation methods, one-pass and sequential. Within each class are a number of variations to meet specific needs. For example, if the experimental data is considered to be perfect and the temperature is measured at every nodal location, a direct deterministic approach can be utilized. On the other hand, if sparsity of temperature measurements can be expected, a direct deterministic approach would not be suitable. Noisy measurements present additional complexities that cannot be accommodated by all of the techniques.

In the presentation to follow, the correlation methods are grouped under the headings, one-pass and sequential methods. Within each class are various methods which are discussed in some detail individually.

3.1 ONE-PASS METHODS

These methods involve the estimation of parameters and states at some epoch whereby all the measurements are processed in one pass; several of these methods are presented below.

3.1.1 Deterministic (Perfect Measurements and No-Sparsity)

The deterministic method requires accurate (if not perfect) temperature measurements at each and every nodal location; if transient temperatures are used, the temperature derivative must also be known accurately. If the number of unknown parameters (excluding capacitances) are less than or equal to the number of nodal temperatures, steady state heat balance equations may be used; otherwise transient temperature data must be utilized.

Consider the model heat balance equations (2-1); if the measured temperatures were used in lieu of model temperatures, the heat flow at the ith node would be zero only if the measured temperatures and the coefficients were perfect. If all of the coefficients were not exact, the net heat flux at the ith node would be a finite value, $\tau_i(t)$, Equation 2-1 would be expressed as:

$$\tau_i(t) = C_i \frac{dT_i}{dt} - Q_i + \sum_{j=1}^{r} a_{ij}(T_i - T_j) + \sum_{j=1}^{r} \sigma b_{ij}(T_i^4 - T_j^4) \qquad (3\text{-}1)$$

$$i = 1, 2, \ldots\ldots, n$$
$$T_j = \text{constant for } n < j \leq r$$
$$a_{ij} = a_{ji}$$
$$b_{ij} = b_{ji}$$

where all the coefficients are not exact

This means by setting $\tau_i(t) = 0$ and using the temperature data at a sufficient number of time instants, a set of linear equations can be generated to obtain the parameter values; this method assumes, of course, that the temperature derivatives can be determined with arbitrary accuracy and, hence, can be assumed to be perfect. Note that since multiplication of each parameter by a constant for fixed i gives the same equation, only ratios of the parameters to one parameter can be determined for each node. It should also be noted that if the time slices are too closely spaced, the parameter coefficient matrix will be almost singular. That is the set of equations are ill-conditioned; this leads to numerical solution difficulties.

If the temperature data are not accurate, the parameter values would necessarily be forced to compensate for the inaccuracies. The effects of small temperature measurement errors are discussed in Appendix D.

3.1.2 Method of Least Squares (Net Heat Flux Residual)

One formulation of least squares is the use of the net heat flux residual at every node, i, as indicated in equation (3-1). This method also requires that the temperatures be measured at every nodal location. If the temperature at every node were perfect, then the residual at a given node is due to the inaccuracies of the parameter values. If the temperature measurements were not accurate, then the heat flux residual represents combined effects of temperature and parameter inaccuracies.

Using equation (3-1), the mean squared residue, R, may be expressed over a time period, $t_f - t_o$, as follows:

$$R = \frac{1}{(t_f - t_o)} \int_{t_o}^{t_f} \sum_{i=1}^{n} [\tau_i(t)]^2 \, dt \qquad (3\text{-}2)$$

The set of normal equations are obtained by taking the derivative of equation (3-1) with respect to the parameters, Q_k, a_{kl}, b_{kl}, C_k, and T_k. The normal equations are easily found and are expressed in the following way.

Derivative with respect to Q_k, yields

$$\frac{\partial R}{\partial Q_k} = \frac{2}{(t_f - t_o)} \int_{t_o}^{t_f} \tau_k(t) \frac{\partial \tau_k(t)}{\partial Q_k} \, dt = 0 \qquad (3\text{-}3)$$

$$k = 1,2,.....,n$$

where: $\dfrac{\partial \tau_k(t)}{\partial Q_k} = 1$

Thus, equation (3-3) becomes,

$$\frac{\partial R}{\partial Q_k} = \frac{2}{(t_f - t_o)} \int_{t_o}^{t_f} \tau_k(t) \, dt = 0 \qquad (3\text{-}4)$$

$$k = 1,2,.....,n$$

Derivative with respect to $a_{k\ell}$ yields,

$$\frac{\partial R}{\partial a_{k\ell}} = \frac{2}{(t_f - t_o)} \int_{t_o}^{t_f} [\tau_k(t) \frac{\partial \tau_k(t)}{\partial a_{k\ell}} + \tau_\ell(t) \frac{\partial \tau_\ell(t)}{\partial a_{k\ell}}] \, dt = 0 \qquad (3\text{-}5)$$

$$k = 1,2,.....,n$$
$$\ell = k+1,.....,r$$

where, $\dfrac{\partial \tau_k(t)}{\partial a_{k\ell}} = T_\ell - T_k$

$$\frac{\partial \tau_\ell(t)}{\partial a_{k\ell}} = T_k - T_\ell$$

Thus, equation (3-4) becomes

$$\frac{\partial R}{\partial a_{k\ell}} = \frac{2}{(t_f - t_o)} \int_{t_o}^{t_f} [\tau_k(t) - \tau_\ell(t)][T_\ell - T_k]\, dt = 0 \qquad (3\text{-}6)$$

$$k = 1,2,.....,n$$

$$\ell = k+1,.....,r$$

Derivative with respect to $b_{k\ell}$ yields,

$$\frac{\partial R}{\partial b_{k\ell}} = \frac{2}{(t_f - t_o)} \int_{t_o}^{t_f} [\tau_k(t) \frac{\partial \tau_k(t)}{\partial b_{k\ell}} + \tau_\ell(t) \frac{\partial \tau_\ell(t)}{\partial b_{k\ell}}]\, dt = 0 \qquad (3\text{-}7)$$

$$k = 1,2,.....,n$$

$$\ell = k+1,.....,r$$

where, $\frac{\partial \tau_k(t)}{\partial b_{k\ell}} = \sigma(T_\ell^4 - T_k^4)$

$$\frac{\partial \tau_\ell(t)}{\partial b_{k\ell}} = \sigma(T_k^4 - T_\ell^4)$$

Thus equation (3-7) becomes

$$\frac{\partial R}{\partial b_{k\ell}} = \frac{2}{(t_f - t_o)} \int_{t_o}^{t_f} [\tau_k(t) - \tau_\ell(t)]\, \sigma[T_\ell^4 - T_k^4]\, dt = 0 \qquad (3\text{-}8)$$

$$k = 1,2,.....,n$$

$$\ell = k+1,.....,r$$

Derivative with respect to C_k yields

$$\frac{\partial R}{\partial C_k} = \frac{2}{(t_f - t_o)} \int_{t_o}^{t_f} [\tau_k(t) \frac{\partial \tau_k(t)}{\partial C_k}]\, dt = 0 \qquad (3\text{-}9)$$

$$k = 1,2,.....,n$$

where, $\frac{\partial \tau_k(t)}{\partial C_k} = -\dot{T}_k$

Thus equation (3-8) becomes

$$\frac{\partial R}{\partial C_k} = \frac{2}{(t_f - t_o)} \int_{t_o}^{t_f} -\tau_k(t)\, \dot{T}_k \, dt = 0 \qquad (3\text{-}10)$$

$$k = 1,2,.....,n$$

Equations (3-4), (3-6), (3-8), and (3-10) represent the normal equations. Solution of these equations will yield the unknown parameter values if the temperatures are known accurately at every node. Since this least squares method is in essence smoothing the information from time, t_o, to, t_f, the problems of accurate temperature derivatives and ill-conditioned equations are considered to be not as severe as for the deterministic method discussed above. It should be reiterated that this particular least squares formulation will not accommodate temperature sparsity which is a major requirement in the present study.

3.1.3 Method of Least Squares (Measured and Model Temperature Difference)[17,18,19]

Another formulation of the least squares method is the use of the temperature difference between measurement and model. With this formulation, random noise can be superimposed on the temperature measurement and temperature sparsity can be accommodated provided of course that physical considerations are satisfied.

Consider a rearranged form of the heat balance equation,

$$\frac{dT_i}{dt} = \frac{Q_i(t)}{C_i} + \sum_{j=1}^{r} \frac{a_{ij}}{C_i} (T_j - T_i) + \sigma \sum_{j=1}^{r} \frac{b_{ij}}{C_i} (T_j^4 - T_i^4) \qquad (3\text{-}11)$$

$$i = 1,2,.....,n$$

If the heat input to each node is a function of time, we have in addition n heat input equations of the form,

$$Q_i(t) = f_i(T_i, T, t, \beta) \qquad (3\text{-}12)$$

where β represents a heating constant such as the solar constant

For a typical thermal model with n nodes, with (p-n) model parameters and m measured nodal temperature, where $m \leq n$ and p is the sum of the number of nodes and the number of model parameters, it is desired to

obtain the least squares estimate of the n nodal temperatures and the (p-n) model parameters at some epoch (time) using the m noise corrupted nodal temperature measurements.

Let the measurement vector, $\{y^*\}_t$, at time t be an m by 1 vector, whose elements are the m noise corrupted measured temperatures at time t,

$$\{y^*\}_t^T = (T^*_{1,t}, T^*_{2,t} \cdots\cdots, T^*_{m,t}) \tag{3-13}$$

(Note the superscript T indicates transpose)

Let the state vector, $\{x\}_t$ at time t be a p by 1 vector, whose elements are the n nodal temperatures and the (p-n) model parameters at time t.

$$\{x\}_t^T = [T_{1,t}, T_{2,t} \cdots\cdots, T_{n,t}, (\frac{Q_i}{C_i})_t, \cdots\cdots, (\frac{b_{ij}}{C_i})_t] \tag{3-14}$$

The relation between the measurement vector and the state vector at time t is given by the following observation equation.

$$\{y^*\}_t = [M]_t \{x\}_t + \{\text{noise}\} \tag{3-15}$$

where $[M]_t$ is the m by p measurement matrix evaluated at time t,

$$[M]_t = \left[\begin{array}{c|c} I & 0 \\ (m \times m) & \end{array}\right] \tag{3-16}$$

Assuming that linear relations* can be used to express the updating of the n nodal temperatures and the n nodal heat input from time t to a later time t+1, and assuming the remaining (p-2n) model parameters are constant, we have the following updating equation for the state vector,

$$\{x\}_{t+1} = [U]_t \{x\}_t \tag{3-17}$$

where $[U]_t$ is the updating matrix evaluated at time t.

---

* The linear relations can usually be obtained by proper linearization of the heat balance equations and the equations governing the heat input, e.g., truncated Taylor Series expansion. Other forms of linearizations lead to different algorithms such as quasilinearization (paragraph 3.1.4) and that used in Program MAFIA (paragraph 3.1.5).

Assuming that k sets of noisy temperature measurements were obtained for the m nodes from time t=0 to time t=k, and these k sets are $\{y^*\}_o$, $\{y^*\}_1,\ldots,\{y^*\}_k$, we wish to obtain the least squares estimate of $\{x\}_k$, whose elements are the n nodal temperatures and the (p-n) model parameters at time t=k. In other words, we want to obtain $x_{i,k}$, the elements of $\{x\}_k$ as a function of the k sets of temperature measurements such that they minimize the following criterion function,

$$\varepsilon_k = \sum_{t=o}^{k} || [M]_t\{x\}_t - \{y^*\}_t ||^2_{[W]_t} \tag{3-18}$$

where $[W]_t$, an m by m diagonal matrix, is an arbitrary positive weighting matrix for the measurement vector $\{y^*\}_t$. This implies that we have some idea of the relative magnitudes of the random measurement noises involved, otherwise $[W]_t$ can be taken to be an identity matrix.

The criterion function ε_k can be regarded as a function of $\{x\}_k$ because equation (3-17) can be used to express $\{x\}_t$ as a function of $\{x\}_k$, $t < k$. Using equation (3-17), we get

$$\{x\}_k = [U]_{k-1}[U]_{k-2}\cdots\cdots[U]_t\{x\}_t \tag{3-19}$$

and

$$\{x\}_t = [\phi]^{-1}_{k,t}\{x\}_k \tag{3-20}$$

where $[\phi]_{k,t}$ is the transition matrix that projects the state vector at time t to the time t=k, and

$$[\phi]_{k,t} \triangleq \prod_{i=t}^{k=1} [U]_i \tag{3-21}$$

Thus letting

$$[K]_{k,t} \triangleq [M]_t[\phi]^{-1}_{k,t} \tag{3-22}$$

Equation (3-18) can be written as a function of $\{x\}_k$ only,

$$\varepsilon_k = \sum_{t=o}^{k} || [K]_{k,t}\{x\}_k - \{y^*\}_t ||^2_{[W]_t} \tag{3-23}$$

The estimation problem is now reduced to the minimization of ε_k with respect to $\{x\}_k$. The set of optimal $\{x\}_k$ that minimize the criterion function ε_k is called the least squares estimate of $\{x\}_k$ and is denoted $\{\hat{x}\}_k$.

Differentiating equation (3-22) with respect to $\{x\}_k$ gives

$$\frac{\partial \varepsilon_k}{\partial \{x\}_k} = 2 \sum_{t=o}^{k} ([K]_{k,t}\{x\}_k - \{y^*\}_t)^T [W]_t [K]_{k,t}$$

and $\{\hat{x}\}_k$ satisfies the following normal equations,

$$\sum_{t=o}^{k} ([K]_{k,t}\{\hat{x}\}_k - \{y^*\}_t)^T [W]_t [K]_{k,t} = 0 \qquad (3\text{-}24)$$

The root, $\{\hat{x}\}_k$, is found by taking the transpose of equation (3-24). This gives,

$$\sum_{t=o}^{k} [([K]_{k,t}\{\hat{x}\}_k - \{y^*\}_t)^T [W]_t [K]_{k,t}]^T = 0$$

which reduces to,

$$\sum_{t=o}^{k} [K]_{k,t}^T [W]_t^T ([K]_{k,t}\{\hat{x}\}_k - \{y^*\}_t) = 0$$

Rearranging, we get

$$\sum_{t=o}^{k} [K]_{k,t}^T [W]_t^T [K]_{k,t}\{\hat{x}\}_k = \sum_{t=o}^{k} [K]_{k,t}^T [W]_t^T \{y^*\}_t$$

Solving for $\{\hat{x}\}_k$ gives,

$$\{\hat{x}\}_k = \left[\sum_{t=o}^{k} [K]_{k,t}^T [W]_t^T [K]_{k,t} \right]^{-1} \left[\sum_{t=o}^{k} [K]_{k,t}^T [W]_t^T \{y^*\}_t \right] \qquad (3\text{-}25)$$

Since $[W]_t$ is a diagonal matrix, and $[W]_t^T = [W]_t$, equation (3-25) can be written as

$$\{\hat{x}\}_k = \left[\sum_{t=o}^{k} [K]_{k,t}^T [W]_t [K]_{k,t} \right]^{-1} \left[\sum_{t=o}^{k} [K]_{k,t}^T [W]_t \{y^*\}_t \right] \qquad (3\text{-}26)$$

Equation (3-26) gives the least squares estimate of the n nodal temperatures and the (p-n) thermal model parameters at time t=k using the noisy temperature measurements for time t=o to time t=k.

If a priori estimates are available, they can be incorporated into the criterion function as additional measurements. Let the a priori

estimate to the state vector at time t=k be $\{x_a\}_k$. Incorporating this a priori estimate into the criterion function as defined by equation (3-23), we get a new criterion function given by,

$$\varepsilon_{k,a} = || \{x\}_k - \{x_a\}_k ||^2_{[A]} + \sum_{t=o}^{k} || [K]_{k,t}\{x\}_k - \{y^*\}_t ||^2_{[W]_t} \tag{3-27}$$

where the p by p matrix, [A], is a weighting matrix for the a priori estimates.

Upon differentiating $\varepsilon_{k,a}$ with respect to the state vector $\{x\}_k$, and solving for the least squares estimate $\{\hat{x}\}_k$ from the resultant normal equations, we get,

$$\{\hat{x}\}_k = [[A] + \sum_{t=o}^{k} [K]^T_{k,t}[W]_t[K]_{k,t}]^{-1}[[A]\{x_a\}_k + \sum_{t=o}^{k} [K]^T_{k,t}[W]_t\{y^*\}_t] \tag{3-28}$$

Equation (3-28) gives the least squares estimate of the n nodal temperatures and the (p-n) thermal model parameters at time t=k using the noisy temperature measurement from time t=o to time t=k and the a priori estimates at time t=k.

3.1.4 Method of Quasilinearization[24,25]

The method of quasilinearization as applied to thermal model parameter correction may be developed as follows:

Consider the heat balance equation (3-11). This set of n equations can be represented by the following vector differential equation,

$$\dot{\underline{T}} = \underline{f}(\underline{T},\underline{p},t) \tag{3-29}$$

where $\underline{T}$ is a vector that consists of the n nodal temperatures and $\underline{p}$ is a vector that consists of the k thermal model parameters. By making transient temperature measurement of all or part of the n isothermal nodes, it is desired to determine the vector $\underline{p}$ and a complete set of initial condition $\underline{T}(t_o)$, for the n nodes, such that the solution to equation (3-29) is in closest agreement with the measured data. Our aim is to determine the thermal model parameter vector $\underline{p}$ and the initial temperature vector $\underline{T}(t_o)$ such that the sum of the squares of the deviations between the computed temperatures and the measured temperatures for the measured nodes is minimized.

Let the mth approximated solution to equation (3-29) be given by

$$\underline{T}^{(m)} = \underline{g}(\underline{T}^{(m)}(t_o),\ \underline{p}^{(m)},\ t) \tag{3-30}$$

By the method of quasilinearization, the m+1 approximated solution can be obtained by solving the following n linearized differential equations

$$\dot{T}_i^{(m+1)} = f_i(\underline{T}^{(m)},\ \underline{p}^{(m)},\ t) + \sum_{j=1}^{n} \frac{\partial f_i(\underline{T}^{(m)}, \underline{p}^{(m)},\ t)}{\partial T_j} (T_j^{(m+1)} - T_j^{(m)}) + \sum_{j=1}^{k} \frac{\partial f_i(\underline{T}^{(m)},\ \underline{p}^{(m)},\ t)}{\partial p_j} (p_j^{(m+1)} - p_j^{(m)}) \qquad i = 1,2,\ldots,n \tag{3-31}$$

The improved thermal model parameter vector $\underline{p}^{(m+1)}$ and the improved initial temperature vector $\underline{T}(t_o)^{(m+1)}$ are determined in the least squares sense by minimizing the following criterion function

$$\epsilon = \sum_{t=o}^{t} || \underline{T}^*(t) - \underline{T}(t)^{(m+1)} ||_{W(t)} \tag{3-32}$$

where $\underline{T}^*(t)$ = measured temperature for the measured nodes

$\underline{T}(t)^{(m+1)}$ = computed temperature for the measured nodes obtained by solving the system of differential equations as given by equation (3-31).

$W(t)$ = some weighting function for the measured temperature if the relative magnitude concerning the measurement noise is known.

Since $\underline{T}(t)^{(m+1)}$ is the solution of a system of linear differential equation, it may be represented in the form,

$$\underline{T}(t)^{(m+1)} - \underline{c}(t,\underline{p}^{(m+1)}) + \sum_{j=1}^{n} \underline{h}_j(t) T_j(t_o)^{(m+1)} \tag{3-33}$$

where the vector $\underline{c}(t,\underline{p}^{(m+1)})$ is the particular solution of equation (3-31) that involves terms consisting of the improved parameter vector $\underline{p}^{(m+1)}$. The vector $\underline{h}_j(t)$ is the solution of the homogeneous form of equation (3-31). These vectors, $\underline{c}(t,\underline{p}^{(m+1)})$ and $\underline{h}_j(t)$, are determined computationally on the measurement interval, time t_o to time t.

The improved initial nodal temperatures, $\underline{T}(t_o)^{(m+1)}$, and the improved thermal model parameters, $\underline{p}^{(m+1)}$, are determined by minimizing the following,

$$\varepsilon = \sum_{t=t_o}^{t} || \underline{T}^*(t) - \underline{c}(t,\underline{p}^{(m+1)}) - \sum_{j=1}^{n} \underline{h}_j(t) T_j(t_o)^{(m+1)} ||_{W_t} \qquad (3\text{-}34)$$

In this way, the problem of thermal model parameters correction is reduced to the solution of systems of linear initial value problems and of linear algebraic equations.

The following summarizes the computational procedures:

(1) Use some nominal parameter values, $\underline{p}^\circ$, and integrate the heat balance equations to get the initial temperature approximation, $\underline{T}^\circ(t)$, by choosing initial conditions arbitrary;

(2) Use $\underline{T}^\circ(t)$ and $\underline{p}^\circ$ to integrate the linearized equations to obtain $\underline{T}^{(1)}(t)$ as a function of the improved parameter vector, $\underline{p}^{(1)}$, and initial temperature vector, $\underline{T}^{(1)}(t_o)$;

(3) Determine $\underline{p}^{(1)}$ and $\underline{T}^{(1)}(t_o)$ in the least squares sense;

(4) Use $\underline{p}^{(1)}$ and $\underline{T}^{(1)}(t)$ to integrate the linearized equations to obtain $\underline{T}^{(2)}(t)$ as a function of the improved parameter vector, $\underline{p}^{(2)}$, and initial temperature vector, $\underline{T}^{(2)}(t_o)$;

(5) Determine $\underline{p}^{(2)}$ and $\underline{T}^{(2)}(t_o)$ in the least squares sense; and

(6) Repeat the iteration until convergence is obtained, i.e.,

$$|\underline{p}^{(m)} - \underline{p}^{(m+1)}| \leq \varepsilon_1$$

$$|\underline{T}^{(m)}(t_o) - \underline{T}^{(m+1)}(t_o)| \leq \varepsilon_2$$

ε_1 and ε_2 are predetermined convergence criteria.

3.1.5 Program MAFIA

The Program MAFIA is a minimum variance estimator whereby the state vector is estimated in the least squares sense. The epoch (time) used is the initial time, and the estimated state vector corresponds to the value at the initial time. The program is programmed to solve three mathematical problems:

(1) The integration of differential equations;

(2) Minimum variance estimation using measured data; and

(3) Solution of nonlinear implicit simultaneous equations.

To correct thermal model parameters using noisy temperature measurements, the nonlinear heat balance differential equations is first integrated using nominal parameter values and initial conditions to obtain nominal temperature profiles. These nominal temperature profiles and the nominal parameter values are then used to linearize the nonlinear heat balance equation.

Let the nonlinear heat balance equations be represented by the following nonlinear vector differential equations,

$$\dot{\underline{x}} = f(\underline{x},t) \tag{3-35}$$

where the state vector $\underline{x}$ is composed of the nodal temperatures and the unknown thermal model parameters. Taking the variation of equation (3-35), we get

$$\delta\dot{\underline{x}} = \frac{\partial f}{\partial \underline{x}} \delta\underline{x} \tag{3-36}$$

Equation (3-36) is a linear equation. The solution to equation (3-35) can be approximated by

$$\underline{x}(t) = \underline{x}^{\circ}(t) + \delta\underline{x}(t) \tag{3-37}$$

where $\underline{x}^{\circ}(t)$ is the nominal state vector obtained by solving equation (3-35) using nominal parameter values and initial conditions. $\delta\underline{x}(t)$ is a correction term and constitutes the solution to equation (3-36); $\delta\underline{x}(t)$ is given by,

$$\delta\underline{x}(t) = \phi(t,t_o)\ \delta\underline{x}(t_o) \tag{3-38}$$

where $\phi(t,t_o)$ is the state transition matrix that projects $\delta\underline{x}$ from time t_o to time t. The state transition matrix is given by,

$$\phi(t,t_o) = \prod_{i=t_o}^{t-1} U_i \tag{3-39}$$

and U_i is the state updating matrix that projects $\delta\underline{x}$ from time t-1 to time t. The state transition matrix and the updating matrix obey the following matrix differential equations,

$$\frac{d}{dt}\phi(t,t_o) = \frac{\partial f}{\partial \underline{x}}\phi(t,t_o) \tag{3-40}$$

$$\phi(t_o,t_o) = I \tag{3-41}$$

$$\frac{d}{dt} U_t = \frac{\partial f}{\partial \underline{x}} U_t \tag{3-42}$$

$$U_{t-1} = I \tag{3-43}$$

Now a matrix equation of the form,

$$\frac{d[W]}{dt} = [A][W] \tag{3-44}$$

with initial conditions

$$[W(t_o)] = [W]_o \tag{3-45}$$

(where [A] is a matrix of constant coefficients), has the solution

$$[W] = [W]_o \, e^{[A](t-t_o)} \tag{3-46}$$

The matrix $e^{[A](t-t_o)}$ may be expressed in an infinite series

$$e^{[A](t-t_o)} = I + \sum_{j=1}^{\infty} \frac{1}{j} A^j (t-t_o)^j \tag{3-47}$$

If the partial matrix $\frac{\partial f}{\partial x}$ is sufficiently constant over the interval t_i-1 to t_i so that the matrix [A] may be approximated by the expression:

$$A \overset{\sim}{=} \frac{1}{2} \left\{ \left(\frac{\partial f}{\partial x}\right)_{t_{i-1}} + \left(\frac{\partial f}{\partial x}\right)_{t_i} \right\} \tag{3-48}$$

and that the matrix $A(t_i - t_{i-1})$ has elements sufficiently small compared to unity that the series expansion will converge using only the first four terms. Thus the updating matrix may be expressed as,

$$\frac{U_t}{U_{t-1}} = I + \sum_{j=1}^{3} \frac{1}{j} (\Delta t)^j \left(\frac{\left(\frac{\partial f}{\partial x}\right)_t + \left(\frac{\partial f}{\partial x}\right)_{t-1}}{2} \right)^j \tag{3-49}$$

where $U_{t-1} = I$ (from equation (3-43))

The state transition matrix may be expressed as

$$\phi(t,t_o) = U_{t-1}\,\phi(t-1,t_o) \tag{3-50}$$

since,

$$\phi(t,t_o) = \prod_{i=t_o}^{t-1} U_i$$

$$= U_{t-1} \prod_{t_o}^{t-2} U_i \tag{3-51}$$

and

$$\prod_{t_o}^{t-2} U_i = \phi(t-1,t_o)$$

With the determination of the state transition matrix and upon substituting equation (3-38) into equation (3-37), we obtain,

$$\underline{x}(t) = \underline{x}^{\circ}(t) + \phi(t,t_o)\delta\underline{x}(t_o) \tag{3-52}$$

and $\delta\underline{x}(t_o)$ can be determined in the least squares sense by minimizing the square of the difference between the estimated temperature and the measured temperature is given by the following criteria function,

$$\varepsilon = \sum_{t_i=o}^{k} || \underline{y}^*(t_i) - \underline{x}(t_i)||^2_{W(t_i)} \tag{3-53}$$

where $\underline{y}^*(t_i)$ is the measurement vector whose elements are the noise corrupted measured temperatures at time t_i and $W(t_i)$ is the measurement weighting matrix.

Upon substituting equation (3-52) into above, we get

$$\varepsilon = \sum_{t_i=o}^{k} || \underline{y}^*(t_i) - \underline{x}^{\circ}(t_i) - \phi(t_i,t_o)\ \delta\underline{x}(t_o)||^2_{W(t_i)} \tag{3-54}$$

Differentiating ε as given by above equation (3-54) with respect to $\delta\underline{x}(t_o)$ and solving for the least squares estimate, $\delta\hat{\underline{x}}(t_o)$, from the resultant normal equations, we obtain a formulation which is similar to equation (3-26) presented in a previous paragraph (3.1.3) on a general least squares method.

$$\delta\underline{\hat{x}}(t_o) = \left[\sum_{t_i=o}^{k} \phi(t_i,t_o)^T W(t_i)\ \phi(t_i,t_o)\right]^{-1} \qquad (3\text{-}55)$$

$$\left[\sum_{t_i=o}^{k} \phi(t_i,t_o)^T W(t_i)\{y^*(t_i)-\underline{x}^\circ(t_i)\}\right]$$

Equation (3-55) yields the correction to the unknown thermal model parameters and the initial nodal temperatures.

When correction to the thermal model parameters and the initial nodal temperatures are obtained, these improved thermal model parameters and initial conditions are than used to integrate the heat balance equations to obtain new nominal temperature profiles and the whole correction problem is rerun. This continues (usually several iterations) until convergence is obtained, i.e.,

$$\left| \frac{\delta\hat{x}(t_o)}{x(t_o)} \right| \leq \Delta$$

Δ is some predetermined convergence criterion.

3.1.6 Maximum Likelihood Estimation[1,6,17]

The maximum likelihood method operates on a parameter or variable which appears in the probability distribution function in such a way that the value of the parameter or variable maximizes the likelihood function. A likelihood function is the probability function when regarded as a function of the parameter.

For example, suppose y^k is a set of data from which a set of parameters θ are to be constructed. Suppose also the joint probability of y^k, given θ, $p(y^k/\theta)$, is available where θ appears as the parameter. When θ are regarded as the variables, the $p(y^k/\theta)$ is a likelihood function. The maximum likelihood estimate of θ, θ^*, is given by the relation,

$$p(y^k/\theta^*) = \max_{\theta} p(y^k/\theta) \qquad (3\text{-}56)$$

Consider a static system.

$$Y_i = H_i X + \eta_i \qquad (3\text{-}57)$$

where η_i's is the m-dimensional Gaussian random noise vector,

$E(\eta_i) = 0$, $E(\eta_i\ \eta_j^T) = R_i\delta_{ij}$

δ_{ij} is the Kronecker delta

where R_i is an m x m noise covariance matrix.

The probability function for η_i is given by,

$$p(\eta_i) = \frac{1}{(2\pi)^{m/2}|R_i|^{1/2}} \exp. \left(-\frac{1}{2}\eta_i^T R_1^{-1}\eta_i\right) \tag{3-58}$$

where $|R_i|$ is the determinant and the joint probability function of $\eta_o,\ldots.\eta_k$ is given by

$$p(\eta_o,\ldots.\eta_k) = (2\pi)^{(-1/2)m(k+1)} \prod_{i=o}^{k} |R_i|^{-1/2} \exp. \left\{-\frac{1}{2}\sum_{i=o}^{k} \eta_i^T R_i^{-1}\eta_i\right\} \tag{3-59}$$

We can write explicitly the probability function of the Gaussian random variable η given by equation (3-58), as soon as we know its mean value and the covariance matrix, because these two quantities are sufficient statistics and serve as the parameters which specify the probability function uniquely.

By changing the variable from η's to Y's in equation (3-59), we get

$$p(y_o,y_1,\ldots.y_k/x) = (2\pi)^{(-1/2)m(k+1)} \prod_{i=o}^{k} |R_i|^{-1/2} \exp. \left\{-\frac{1}{2}\sum_{i=o}^{k} ||H_i x-y_i||^2_{R_i^{-1}}\right\} \tag{3-60}$$

In the above equation, the variables to be estimated, x, appears as the parameters vector of the joint probability function. This is exactly the likelihood function for the observations $y_o,\ldots.y_k$. The maximum likelihood estimate of x, x_k^*, is the x which maximize equation (3-60) or which minimize the term

$$\sum_{i=1}^{k} ||H_i x-y_i||^2_{R_i^{-1}} \tag{3-61}$$

By identifying the weighting function is a least squares estimator with R_i^{-1}, we see that the least squares estimator is the same as a maximum likelihood estimator.

3.1.7 Regression Analysis[1,2,3]

Regression analysis is a means of making an interpretation of the outcome of an experiment. This is done by a mathematical formulation that usually consists of expressing the expected value of one variable (designated as the dependent variable) as a function of the observed values of other variables (designated as the independent variables) and unknown parameters called regression parameters. The regression parameters are determined by using the observed values of the independent variables and dependent variable in a least squares sense.

For a linear regression model, the dependent and independent variables are related by a linear relationship of the following form.

$$y = \beta_o + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \ldots\ldots \beta_n x_n \qquad (3\text{-}62)$$

where y is the dependent variable

x_i are the independent variables, $i = 1,\ldots\ldots n$

β_i are the regression parameters, $i = 0,\ldots\ldots n$

If m sets of data are obtained for y and the x_i's, and $m > n$, then the regression parameters can be obtained in the least squares sense, by solving the following sets of normal equations,

$$\frac{\partial \rho}{\partial \beta_i} = \frac{\partial}{\partial \beta_i} \sum_{k=1}^{m} \left(y_k - \sum_{i=o}^{n} \beta_i x_{i,k}\right)^2 = 0 \qquad (3\text{-}63)$$

where $x_{o,k} = 1$

Consider the heat balance equation (3-11) for the ith node of a n node model,

$$\dot{T}_i = \frac{Q_i}{C_i} + \sum_{j=1}^{n} \frac{a_{ij}}{C_i} (T_j - T_i) + \sum_{j=1}^{n} \frac{\sigma b_{ij}}{C_i} (T_j^4 - T_i^4)$$

If the time histories of $\dot{T}_i$, T_j and T_i are known, one can regard equation (3-11) as a linear regression model by itself independent of the remaining heat balance equations; one would regard $\dot{T}_i$ as the dependent variable, $(T_j - T_i)$ and $(T_j^4 - T_i^4)$ as the independent variables, and the $\frac{Q_i}{C_i}$, $\frac{a_{ij}}{C_i}$, and $\frac{\sigma b_{ij}}{C_i}$ as the regression parameters. These

regression parameters can be determined in the least squares sense by solving a set of normal equations similar to equation (3-63) in form.

Once the parameters in the ith heat balance equation is determined, they can be substituted into the jth heat balance equation where some of these parameters appears, and the remaining unknown parameters in the jth equations can then be determined. By taking the heat balance equation one at a time, it is possible to determine all the unknown parameters in the thermal model sequentially.

Note that the term sequential as used here is taken in the equation-wise sense whereas the one-pass and sequential correlation methods are distinguished in the timewise sense.

The inherent advantage of the regression method is that only a single heat balance equation need be considered at a time; this reduces considerably the size of the core storage requirement relative to methods that require simultaneous solutions of the heat balance equations. The major problems with the regression method are that temperature sparsity cannot be handled and round-off errors could propagate as the heat balance equation is being processed sequentially with the result that large errors could be introduced into the equations that are processed near and at the end.

3.2 SEQUENTIAL METHODS

The sequential methods involve the estimation of parameters and states sequentially, whereby the measurement data are processed sequentially and new estimates of the parameters are obtained after each set of measurements is processed. Several of these methods are presented below:

3.2.1 Kalman Filtering Method[26,27,28,29]

Again as assumed for the one-pass method, the functional model is considered to represent the physical system accurately and it is desired to correct the model parameters using the measured temperatures. Since the measured temperatures generally include random observation noises, it is desirable formulate a technique whereby the random noises are filtered. Consider the heat balance equation (3-11)

$$\frac{dT_i}{dt} = \frac{Q_i(t)}{C_i} + \sum_{j=1}^{n} \frac{a_{ij}}{C_i} (T_j - T_i) + \sigma \sum_{j=1}^{n} \frac{b_{ij}}{C_i} (T_j^4 - T_i^4) \qquad i = 1,2,.....,n$$

with the heat input function as each node expressed as in equation (3-12)

$$Q_i(t) = f_i(T_i, T_j, t, \beta)$$

An applicable method currently being used in the field of Optimal Control is the method of Kalman filtering. In this study, the method of Kalman filtering is applied to correct the various thermal model parameters using noisy temperature measurements.

For a thermal model that contains n nodes with m nodal temperatures measured, where $m \leq n$, the random noise corrupted measurement vector, $\{y^*\}$, is an m by 1 vector whose elements are given by the m noise corrupted measured temperatures. This is given by

$$\{y^*\}^T = (T_1^* \; T_2^* \; T_3^* \; \ldots\ldots \; T_i^* \; \ldots\ldots \; T_m^*) \tag{3-64}$$

where T_i^* = random noise corrupted measured temperature for the ith node, i = 1,2,.....,m

In the present problem, the sum of the number of model parameters and the number of isothermal nodes is p. The state vector is a p by 1 vector whose elements are the n nodal temperatures and the (p-n) model parameters. The (p-n) model parameters are the $(\frac{Q_i}{C_i})$'s, $(\frac{a_{ij}}{C_i})$'s, and the $(\frac{b_{ij}}{C_i})$'s. The state vector is given by the following,

$$\{x\}^T = (T_1 \; T_2 \; \ldots\ldots T_n, \frac{Q_i}{C_i} \; \ldots\ldots \frac{a_{ij}}{C_i} \; \ldots\ldots \frac{b_{ij}}{C_i}) \tag{3-65}$$

The relation between the measurement vector and the state vector is given by the following observation equation,

$$\{y^*\} = [M]\{x\} + \{W\} \tag{3-66}$$

where [M] is the m by p measurement matrix given by

$$[M] = \left[\begin{array}{c:c} \underset{(m \times m)}{I} & 0 \end{array}\right] \tag{3-67}$$

and $\{W\}$ is the m by 1 random measurement noise vector whose elements are the random noises associated with the m measured temperatures. This is given by

$$\{W\}^T = (W_1 \; W_2 \; \ldots\ldots\ldots W_m) \tag{3-68}$$

It is assumed that $\{W\}$ has a zero mean, that is $E\{W\} = 0$, and the covariance matrix of $\{W\}$ is given by,

$$[W] = E\,[\{W\}\{W\}^T] \tag{3-69}$$

Let $\{x_a\}$ be an unbiased a priori estimate of $\{x\}$, where unbiased means $E\,[\{x\} - \{x_a\}] = 0$, and let the p by p error covariance matrix of $\{x_a\}$ be given as,

$$A = E\,[(\{x\} - \{x_a\})(\{x\} - \{x_a\})^T] \tag{3-70}$$

Furthermore, it is assumed that the random measurement error and the a priori estimation error are uncorrelated, that is

$$E\,[\{W\}\,\{x-x_a\}^T] = 0 \tag{3-71}$$

It is now desired to obtain a new and "better" estimate of $\{x\}$ using the measurement $\{y^*\}$ and the a priori estimate $\{x_a\}$. Let the new estimate of $\{x\}$ be $\{\hat{x}\}$, and let $\{\hat{x}\}$ be a linear combination of $\{y^*\}$ and $\{x_a\}$ such that the following relation is satisfied

$$\{\hat{x}\} = [B]\{y^*\} + [C]\{x_a\} \tag{3-72}$$

The matrix [B] and [C] are called the measurement weighting matrix and the a priori weighting matrix respectively.

The Determination of the Measurement Weighting Matrix and the A Priori Weighting Matrix

For $\{\hat{x}\}$ to be an unbiased estimate, $E\{\hat{x}\}$ must be equal to $E\{x\}$. Thus from equation (3-72), we get

$$E\{\hat{x}\} = E([B]\{y^*\} + [C]\{x_a\}) \tag{3-73}$$

Substituting equation (3-66) into above, we get

$$E\{\hat{x}\} = E([B][M]\{x\} + [B]\{W\} + [C]\{x_a\})$$

which becomes,

$$E\{\hat{x}\} = [B][M]E\{x\} + [B]E\{W\} + [C]E\{x_a\} \tag{3-74}$$

since

$$E\{x\} = E\{x_a\} \text{ and } E\{W\} = 0$$

Equation (3-74) reduces to

$$E\{\hat{x}\} = ([B][M] + [C])E\{x\}$$

Thus in order for $\{\hat{x}\}$ to be an unbiased estimate, such that

$$E\{\hat{x}\} = E\{x\}$$

we must have

$$[B][M] + [C] = [I] \tag{3-75}$$

where [I] is an p byp identity matrix.

Similar to the error covariance matrix for $\{x_a\}$, we define the error covariance matrix of the estimate $\{\hat{x}\}$

$$J = E[\{x-\hat{x}\}\{x-\hat{x}\}^T] \tag{3-76}$$

Using equation (3-72), (3-66), and (3-75), we obtain,

$$\begin{aligned} J &= E[\{x-B\underline{y}^* - C\underline{x}_a\}\{x-By^* - Cx_a\}^T] \\ &= E[\underline{x} - BM\underline{x} - B\underline{W} - C\underline{x}_a\}\{\underline{x} - BM\underline{x} - B\underline{W} - Cx_a\}^T] \\ &= E[\{(I - BM)\underline{x} - B\underline{W} - (I - BM)x_a\}\{(I - BM)\underline{x} - B\underline{W} - (I - BM)x_a\}^T] \\ &= E[\{(I - BM)(\underline{x} - x_a) - BW\}\{(I - BM)(x-x_a) - B\underline{W}\}^T] \end{aligned}$$

With equation (3-69) and (3-70), above equation reduces to

$$J = ([I]-[B][M])[A]([I]-[B][M])^T + ([B][W][B]^T) \tag{3-77}$$

In order to obtain a minimum variance estimate of $\{x\}$, the matrix [B] must be chosen such that the quadratic form associated with J is minimized. The quadratic form of J is given by,

$$S = \{Z\}^T[J]\{Z\} \tag{3-78}$$

where $\{Z\}$ is any arbitrary p by one vector.

Taking the variation of S, gives

$$\delta S = \{Z\}^T\, \delta[J]\{Z\} \tag{3-79}$$

From equation (3-77), we get

$$\delta[J] = [-\delta BMA(I-BM)^T - (I-BM)\, AM^T\, \delta B^T + \delta BWB^T + BW\delta B^T]$$

Upon collecting terms and substituting above into equation (3-79), we get

$$\delta S = \{Z\}^T\, [-(I-BM)AM^T + BW]\, \delta B^T\, \{Z\} + [\{Z\}^T\, [-(I-BM)AM^T + BW]\, \delta B^T\{Z\}]^T$$

Since both terms in the right hand side are scalers, each of which is the transpose of the other, and because the transpose of a scaler is also a scaler, we obtain,

$$\delta S = 2\{Z\}^T [-(I-BM)AM^T + BW] \delta B^T\{Z\}$$

S will be minimized for all $\{Z\}$ if

$$(I-BM)AM^T = BW \tag{3-80}$$

Solving for B gives

$$AM^T - BMAM^T - BW = o$$

and

$$B - AM^T(MAM^T+W)^{-1} \tag{3-81}$$

Combining equations (3-72) and (3-75), we get

$$\{\hat{x}\} = \{x_a\} + [B](\{y^*\}-\{y_a\}) \tag{3-82}$$

The above equation together with equation (3-81), gives the minimum variance estimate $\{\hat{x}\}$ of $\{x\}$.

Upon substitution of equation (3-81) into equation (3-77), the expression for the optimal error covariance matrix is obtained to give,

$$J = (I-BM)A \tag{3-83}$$

The Kalman Filter Equations

The equations developed in the previous two sections form the basis for the Kalman filter equations. The remaining task is to show how $\{x_a\}$ and [A] for a new time $t+\Delta t$ can be obtained from $\{\hat{x}\}$ and [J] for the time t. In other words, one wishes to use estimated at time t to form the a priori estimate for time $t+\Delta t$. The time, t is taken to mean the time at which a measurement or measurements are made, and time, $t+\Delta t$ is taken to mean the time at which the next measurement or measurements are made.

Let the state vector at time $t+\Delta t$ be expressible by some linear function* involving the value of the state vector at time t.

*The linear relations can usually be obtained by proper linearization of the heat balance equations and the equations governing the heat input, e.g., truncated Taylor series expansion.

$$\{x\}_{t+\Delta t} = [U]_t\{x\}_t \tag{3-84}$$

where $[U]_t$ is the updating matrix that projects the state vector at time t to a new time t+Δt.

Using Equation (3-84), the new a priori estimate for the state vector is obtained to give,

$$\{x_a\}_{t+\Delta t} = [U]_t\{\hat{x}\}_t \tag{3-85}$$

and the associated a priori error covariance matrix is given by

$$[A]_{t+\Delta t} = E[(\{x\}_{t+\Delta t} - \{x_a\}_{t+\Delta t})(\{x\}_{t+\Delta t} - \{x_a\}_{t+\Delta t})^T]$$

$$= E[[U]_t\{x\}_t - \{\hat{x}\}_t)(\{x\}_t - \{\hat{x}\}_t)^T[U]_t^T]$$

Using equation (3-76), we get

$$[A]_{t+\Delta t} = [U]_t[J]_t[U]_t^T \tag{3-86}$$

Equations (3-85) and (3-86) give the relationship whereby the new a priori estimate of the state vector and its associated error covariance matrix can be obtained. This completes the derivation of the Kalman filter equation.

Summary

The following summarizes the Kalman filter equations whereby the correction of thermal model parameters can be obtained sequentially.

$$\{y^*\}_t = [M]_t\{x\}_t + \{W\}_t \tag{3-87}$$

$$\{x\}_{t+\Delta t} = [U]_t\{x\}_t \tag{3-88}$$

$$\{\hat{x}\}_t = \{x_a\}_t + [B]_t(\{y^*\}_t - \{y_a\}_t) \tag{3-89}$$

$$\{y_a\}_t = [M]_t\{x_a\}_t \tag{3-90}$$

$$[B]_t = [A]_t[M]_t^T([M]_t[A]_t[M]_t^T + [W]_t)^{-1} \tag{3-91}$$

$$[J]_t = ([I] - [B]_t[M]_t)\ [A]_t \tag{3-92}$$

$$\{x_a\}_{t+\Delta t} = [U]_t\{\hat{x}\}_t \tag{3-93}$$

$$[A]_{t+\Delta t} = [U]_t[J]_t[U]_t^T \tag{3-94}$$

where $\{y^*\}_t$ = random noises corrupted measurement vector obtained at time, t.

$\{x\}_t$ = value of the state vector (unknown parameters) at time, t.

$\{W\}_t$ = random noises associated with the measured data obtained at time, t.

$\{x\}_{t+\Delta t}$ = value of the state vector (unknown parameters) at time, $t+\Delta t$.

$\{M\}_t$ = measurement matrix evaluated at time, t.

$\{\hat{x}\}_t$ = new estimate of the state vector (unknown parameters) after processing the measured data obtained at time, t.

$\{\hat{x}_a\}_t$ = a priori estimate of the state vector (unknown parameters) before processing the measured data obtained at time, t.

$[B]_t$ = measurement weighting matrix evaluated at time, t (the time varying gain).

$[A]_t$ = $E[(\{x\} - \{x_a\})(\{x\} - \{x_a\})^T]$, error covariance matrix for the a priori estimate state vector.

$[J]_t$ = $E[(\{x\} - \{\hat{x}\})(\{x\} - \{\hat{x}\})^T]$, error covariance matrix for the newly estimated state vector.

$[U]_t$ = transition matrix

The following steps given the correction scheme whereby the Kalman filter equations are used,

(1) First obtain an a priori estimate for the state vector $\{x_a\}_t$ and the associated error covariance matrix $[A]_t$.

(2) Calculate the time varying gain $[B]_t$ using equation (3-91) and the first set of measured data.

(3) Obtain new estimate for the state vector, $\{\hat{x}\}_t$ using equation (3-89) and the first set of measured data.

(4) Calculate the error covariance matrix, $[J]_t$ for the newly estimated $\{\hat{x}\}_t$ using equation (3-92).

(5) Update the newly estimated state vector, $\{\hat{x}\}_t$ with equation (3-93) to obtain the new a priori estimate at time, $t+\Delta t$, and calculate its associated error covariance matrix using equation (3-94).

(6) Repeat Step (2) to (5) using the new a priori estimate for the state vector and its associated error covariance matrix with the 2nd set of measured data.

(7) Repeat above until all the measured data have been processed or until desirable results* are obtained.

Formulation When the Model Parameters are Temperature Dependent

For large temperature variation, the thermal properties of many spacecraft materials may vary greatly, and the model parameters become highly temperature dependent. For these temperature dependent parameters, a scheme whereby corrections can be obtained using the equations obtained in the previous sections is formulated as follow.

Consider a typical conductive conductive joining the nodes i and j. This conductance is in general given by the equation,

$$a_{ij} = \frac{k_{ij} A_{ij}}{(\Delta x)_{ij}} \tag{3-95}$$

where k_{ij} = thermal conductivity of the material

A_{ij} = area along the conduction path joining nodes i and j.

$(\Delta x)_{ij}$ = length along the conduction path joining nodes i and j.

The temperature dependence of the thermal conductivity can usually be expressed as

$$k = k^{\circ} f(T) \tag{3-96}$$

where k° = same reference thermal conductivity value

$f(T)$ = a known function in terms of the material temperature**

Substituting equation (3-96) into equation (3-95), we get

$$a_{ij} = \frac{k^{\circ}_{ij} A_{ij}}{(\Delta x)_{ij}} \; f(T_i, T_j) \tag{3-97}$$

From the above equation, it can be observed that the conductive conductance, a_{ij}, is a product of two terms, one being temperature dependent and one being a constant. Let the constant $\frac{k^{\circ}_{ij} A_{ij}}{(\Delta x)_{ij}}$ be a°_{ij}. Equation (3-97) becomes,

*Desirable results are results whose variance are smaller than some specified values.

**For a thermal model, the material temperature is usually expressed as some weighted average of the temperatures at nodes i and j.

$$a_{ij} = a^{\circ}_{ij}\ f(T_i,T_j) \tag{3-98}$$

Similarly, it is possible to express the radiative conductive in the above form, giving

$$b_{ij} = b^{\circ}_{ij}\ g(T_i,T_j) \tag{3-99}$$

where $g(T_i,T_j)$ is some known function relating the radiation properties of nodes i and j to its temperatures.

Using equations (3-98) and (3-99) for the a_{ij}'s and the b_{ij}'s, the heat balance equation for node i can be written as,

$$\frac{dT_i}{dt} = \frac{Q_i}{C_i} + \sum_{j=1}^{n} \left(\frac{a^{\circ}_{ij}}{C_i}\right) f(T_i,T_j)(T_j-T_i) + \sum_{j=1}^{n} \left(\frac{b^{\circ}_{ij}}{C_i}\right) g(T_i,T_j)(T_j^4-T_i^4) \tag{3-100}$$

The above heat balance equation takes the same form as equation (3-11) withe the exception that (T_j-T_i) has been replaced by $f(T_i,T_j)(T_j-T_i)$ and $(T_j^4-T_i^4)$ has been replaced by $g(T_i,T_j)(T_j^4-T_i^4)$. Since the same temperature measurements are used, the equations for parameters correction developed in the previous sections also apply.

Convergence Criterion

The Kalman filter is an unbiased minimum variance linear estimator. By examining the diagonal terms of the error covariance matrix, which give the variances of the model parameters and nodal temperatures, it is possible to determine the convergent trend as the noisy temperature measurements are being processed. Convergent trend results in decreasing variances as additional noisy temperature measurements are processed. Convergent criterion for the model parameters can be defined by setting some lower limits.

Measurement Noise

In the application of the Kalman filtering method to the correction of thermal model parameters, some knowledge regarding the statistics of the measurement noise must be available. For truly random measurement errors, Gaussian noise can probably be assumed. When thermocouple biases are associated with the measurements, and these biases are known, they must be removed such that the only noises associated with the measurements are the random noises. If the thermocouple biases are

unknown, these biases can be considered as additional unknown parameters to be determined and included as part of the state vector.

3.2.2 Recursive Least Squares[16-19]

In paragraph 3.1.3 was presented the one-pass least squares method. Suppose additional temperature measurements are available at a later time, t=k+1, and to obtain the least squares estimate of the n nodal temperatures and the (p-n) thermal model parameters, it is necessary to resolve the entire problem from time t=0 to time t=k+1 if the standard least squares method is used; it is desirable to develop a recursive scheme whereby the optimal least squares estimate can be obtained sequentially whenever additional temperature measurements are available. In the development to follow, continued reference will be made to the presentation of paragraph 3.1.3.

The criterion function to be minimized with temperature measurements from time t=0 to t=k+1 is given by the following expression which is similar to equation (3-18),

$$\varepsilon_{k+1} = \sum_{t=o}^{k+1} \left|\left| [M]_t \{x\}_t - \{y^*\}_t \right|\right|^2_{[W]_t} \tag{3-101}$$

The optimal least squares estimate of $\{x\}_{k+1}$, $\{\hat{x}\}_{k+1}$, is given by (which is similar to equation (3-26)).

$$\{\hat{x}\}_{k+1} = \left[\sum_{t=o}^{k+1} [K]^T_{k+1,t}[W]_t[K]_{k+1,t}\right]^{-1}\left[\sum_{t=o}^{k+1} [K]^T_{k+1,t}[W]_t\{y^*\}_t\right] \tag{3-102}$$

With the definition of [K] and [ϕ] as given by equations (3-22) and (3-21) respectively, we can write,

$$[K]_{k+1,t} = [M]_t[\phi]^{-1}_{k+1,t} \tag{3-103}$$

$$[\phi]_{k+1,t} = [U]_k[\phi]_{k,t} \tag{3-104}$$

$$[K]_{k+1,t} = [M]_t[\phi]^{-1}_{k,t}[U]^{-1}_k = [K]_{k,t}[U]^{-1}_k \tag{3-105}$$

Substituting equation (3-105) into equation (3-102), we get

$$\{\hat{x}\}_{k+1} = \left[\sum_{t=o}^{k+1} [U]_k^{-1^T} [K]_{k,t}^T [W]_t [K]_{k,t} [U]_k\right]^{-1} \left[\sum_{t=o}^{k+1} [U]_k^{-1^T} [K]_{k,t}^T [W]_t \{y^*\}_t\right] \tag{3-106}$$

Equation (3-106) can be expanded to give,

$$\{\hat{x}\}_{k+1} = \left[\sum_{t=o}^{k} [U]_k^{-1^T} [K]_{k,t}^T [W]_t [K]_{k,t} [U]_k^{-1} + [U]_k^{-1^T} [K]_{k,k+1}^T [W]_{k+1} [K]_{k,k+1} [U]_k^{-1}\right]^{-1}$$

$$\left[\sum_{t=1}^{k} [U]_k^{-1^T} [K]_{k,t}^T [W]_t \{y^*\}_t + [U]_k^{-1^T} [K]_{k,k+1}^T [W]_{k+1} \{y^*\}_{k+1}\right] \tag{3-107}$$

We wish to obtain $\{\hat{x}\}_{k+1}$ as a linear combination of $\{\hat{x}\}_k$ and some correction term $\{\Delta x\}_{k+1}$ in the following form,

$$\{\hat{x}\}_{k+1} = [U]_k \{x\}_k + \{\Delta x\}_{k+1} \tag{3-108}$$

To determine the expression for the correction term, we substitute equation (3-108) into equation (3-107) to give

$$\left[[U]_k^{-1^T} \sum_{t=o}^{k} [K]_{k,t}^T [W]_t [K]_{k,t} [U]_k^{-1} + [U]_k^{-1^T} [K]_{k,k+1}^T [W]_{k+1} [K]_{k,k+1} [U]_k^{-1}\right]$$

$$\left([U]_k \{\hat{x}\}_k + \{\Delta x\}_{k+1}\right) = [U]_k^{-1^T} \sum_{t=o}^{k} [K]_{k,t}^T [W]_t \{y^*\}_t + [U]_k^{-1^T} [K]_{k,k+1}^T [W]_{k+1} \{y^*\}_{k+1} \tag{3-109}$$

Expanding above and solving for $\{\Delta x\}_{k+1}$ after the application of equation (3-26), we get

$$\{\Delta x\}_{k+1} = \left[[U]_k^{-1^T} \sum_{t=o}^{k} [K]_{k,t}^T [W]_t [K]_{k,t} [U]_k^{-1} + [U]_k^{-1^T} [K]_{k,k+1}^T [W]_{k+1} [K]_{k,k+1} [U]_k^{-1}\right]^{-1}$$

$$[U]_k^{-1^T} [K]_{k,k+1} [W]_{k+1} (\{y^*\}_{k+1} - [K]_{k,k+1} \{\hat{x}\}_k) \tag{3-110}$$

Let

$$[P]_k^{-1} = \sum_{t=o}^{k} [K]_{k,t}^T [W]_t [K]_{k,t} \tag{3-111}$$

and

$$[P]_{k+1} = \left[[U]_k^{-1^T} \sum_{t=o}^{k} [K]_{k,t}^T [W]_t [K]_{k,t} [U]_k^{-1} + [U]_k^{-1^T} [K]_{k,k+1}^T [W]_{k+1} [K]_{k,k+1} [U]_k^{-1} \right]^{-1} \tag{3-112}$$

then

$$[P]_{k+1} = \left[[U]_k^{-1^T} [P]_k^{-1} [U]_k^{-1} + [U]_k^{-1^T} [K]_{k,k+1}^T [W]_{k+1} [K]_{k,k+1} [U]_k^{-1} \right]^{-1} \tag{3-113}$$

Upon the application of Ho's lemma*,[15] we get

$$[P]_{k+1} = [U]_k [P]_k [U]_k^T - [U]_k [P]_k [U]_k^T [U]_k^{-1^T} [K]_{k,k+1}^T ([K]_{k,k+1} [U]_k^{-1} [U]_k [P]_k [U]_k^T$$

$$[U]_k^{-1^T} [K]_{k,k+1}^T + [W]_{k+1})^{-1} [K]_{k,k+1} [U]_k^{-1} [U]_k [P]_k [U]_k^T \tag{3-114}$$

With equation (3-17), we obtained,

$$\{x\}_k = [U]_k^{-1} \{x\}_{k+1} \tag{3-115}$$

With the definition of the transition matrix as given by equation (3-21), we get

$$[\phi]_{k,k+1} = [U]_k^{-1}$$

and with equation (3-27), we get

$$[K]_{k,k+1} = [M]_{k+1} [U]_k \tag{3-116}$$

Upon substituting equation (3-116) into equation (3-114), we get

$$[P]_{k+1} = [U]_k [P]_k [U]_k^T - [U]_k [P]_k [U]_k^T [M]_{k+1} ([M]_{k+1} [U]_k [P]_k [U]_k^T [M]_{k+1}^T$$

$$+ [W]_{k+1})^{-1} [W]_{k+1} [U]_k [P]_k [U]_k^T \tag{3-117}$$

With equations (3-108), (3-110), and (3-113), we get

$$\{\hat{x}\}_{k+1} = [U]_k \{\hat{x}\}_k + [P]_{k+1} [M]_{k+1}^T [W]_{k+1} (\{y^*\}_{k+1} - [M]_{k+1} [U]_k \{x\}_k) \tag{3-118}$$

---

*Ho's matrix inversion lemma: If $[P]_1^{-1} = [P]^{-1} + [H]_1^T [W]_1 [H]_1$ where [P] and [W] are nonsingular, symmetric, and positive, then

$$[P]_1 = [P] - [P][H]_1^T ([H]_1 [P] [H]_1^T + [W]_1)^{-1} [H]_1 [P]$$

Equations (3-111), (3-117), and (3-118) give the least squares estimates of the n nodal temperatures and the (p-n) thermal model parameters at time, t=k+1, as a function of the least squares estimate at time t=k and some correction term, and these equations formed the recursive least squares estimator.

3.2.3 Bayesian Estimation[6,17,20]

In many estimation problems, it is possible to specify some "criterion function" resulting from incorrect estimation of the system parameters. If the a priori probability function is available not only for the measurement errors but also for the values of the unknown parameter vector, x, then it is possible to obtain an optimal estimate, x, which optimizes this criterion function. Such an estimate is called an optimal Bayesian estimate.

In general, the form of an optimal Bayesian estimate depends upon the form of both the criterion function and the a priori probability distributions of parameters and measurement noise. For the class of estimation problems with quadratic criterion function, an optimal Bayesian estimate is given by choosing x to be the mean of x conditioned on the observation, y^k, i.e., $\hat{x} = E(x/y^k)$. This is true regardless of the probability distributions of the parameters or measurement noise. For the class of problem with criterion function, $E = |x-\hat{x}|$, then the optimal Bayesian estimate is given by choosing $\hat{x}$ to be the medium of $p(x/y^k)$. If the criterion function is to maximize the probability $p(x/y^k)$, then the optimal Bayesian estimate is given by choosing $\hat{x}$ to be the mode of $p(x/y^k)$. However, if the a posteriori probability distribution of x given the measurements y^k, $p(x/y^k)$ is uniform and symmetric, then all three optimal estimates are the same.

Computation of the a posteriori probability distribution, p(x/y)

Consider the problem of estimating x given y and y is given by,

$$y = g(x,\eta) \tag{3-119}$$

where x is the unknown vector and η is the random noise vector.

We wish to compute the a posteriori probability distribution p(x/y).

If equation (119) is invertible for η

$$\eta = f^{-1}(x,y) \tag{3-120}$$

where f^{-1} is the inverse of g, then

$$p(x,y) = p(x,\eta)|J| \tag{3-121}$$

where J is the Jacobian

$$J = \det. \left(\frac{\partial \eta}{\partial y}\right) \tag{3-122}$$

and

$$p(x/y) = \frac{p(x,y)}{p(y)} \tag{3-123}$$

Another method whereby the a posteriori probability distribution can be obtained is through the use of Baye's rule

$$p(x/y) = \frac{p(x)p(y/x)}{p(y)} \tag{3-124}$$

Example:

As an example, consider the linear system given by

$$y = Hx + \eta \tag{3-125}$$

Assume that x and η are dependent such that the joint probability distribution is given by,

$$p(x,\eta) = p(x)p(\eta) \tag{3-126}$$

and p(x) is Gaussian with

$$E(x) = \bar{x}, \quad E(xx^T) = P_o \tag{3-127}$$

and $p(\eta)$ is also Gaussian with

$$E(\eta) = 0, \quad E(\eta\eta^T) = R \tag{3-128}$$

From equations (3-125, (3-127), and (3-128), we see that y is also a Gaussian random vector with

$$E(y) = H\bar{x}, \quad E(yy^T) = HP_oH^T + R \tag{3-129}$$

and

$$p(y) = \left[(2\pi)^{m/2} |HP_oH^T + R|^{1/2} \right]^{-1} \exp.\{-\frac{1}{2}(y-H\bar{x})^T (R+HP_oH^T)^{-1}(y-H\bar{x})$$

Since the Jacobian, $\det. \left(\frac{\partial \eta}{\partial y}\right) = 1$

then

$$p(x,y) = p(x,\eta) = p(x)p(\eta) = p(x)p(y-Hx)$$

and

$$p(x/y) = \frac{p(x)p(y-Hx)}{p(y)}$$

$$p(x/y) = C \exp. \{-\frac{1}{2}[(x-\bar{x})^T P_o^{-1} (x-\bar{x}) + (y-Hx)^T R^{-1}(y-Hx)$$

$$-(y-H\bar{x})^T (R+HP_oH^T)^{-1}(y-H\bar{x})]\} \quad (3\text{-}130)$$

where

$$C = \frac{1}{(2\pi)^{n/2}} \quad \frac{|HP_oH^T + R|^{1/2}}{|P_o|^{1/2}|R|^{1/2}} \quad (3\text{-}131)$$

Since $p(x/y)$ is symmetrical and uniform, all three best estimates, i.e., the conditional mean, median, and mode of $p(x/y)$ are the same and given by x which is given by

$$\hat{x} = \bar{x} + PH^T R^{-1}(y-H\bar{x}) \quad (3\text{-}132)$$

$$p = P_o - P_oH^T(R+HP_oH^T)^{-1}HP_o \quad (3\text{-}133)$$

Equation (3-131) and (3-132) are essentially the Wiener-Kalman filter for a single stage estimation.

3.3 SUMMARY

A number of potentially suitable correlation methods selected from a list of techniques reported in technical literature and categorized as either one-pass or sequential were presented in mathematical detail above. Although each of these methods may potentially be suitable as a technique to fullfil the objectives of the present study, a limited number of methods had to be chosen for further studies because comparison could be obtained only by coding the selected methods and applied to a mathematical model. Since temperature sparsity is a major requirement, it was clear that techniques that could (at least potentially) accommodate sparse measurement must be selected. As a result the Kalman filter which is a sequential method and Program MAFIA and quasilinearization which are one-pass methods were selected as the major techniques to be studied in detail. These methods because of mathematical sophistication are subject to programming difficulties. In order to obtain baseline

information such as core storage requirements and round-off error and to provide comparison results two other techniques, regression analysis and the method of least squares with heat flux residual were selected for additional study. Both of these methods will not accommodate sparse temperature measurement but mathematical sophistication is much less than the aforementioned methods. Evaluation of these techniques are discussed in Section 4.

4.0 EVALUATION OF SELECTED CORRELATION METHODS

Five methods as indicated in the previous section were selected for more detailed study. Two of the methods, regression analysis and least squares (net heat flux residual) were studied to obtain baseline information such as round-off errors, programming complexities, and results for comparison with the other selected methods. Both of these methods will not handle sparsity of temperature measurements, which is a major requirement of this study. The Kalman filter, Program MAFIA, and quasilinearization are the other three methods to be studied. The first is a sequential method and the latter two are one-pass methods; these three methods will accommodate sparsity. Initially, it was the intent to study both Program MAFIA was found to be a satisfactory correlation technique.

Evaluation of these methods were pursued by the use of the five- and twenty-node models which are described in Appendix A. The evaluation of these methods are present below.

4.1 METHOD OF LEAST SQUARES (NET HEAT FLUX RESIDUAL)

This method as discussed in paragraph 3.1.2 is a least squares method of the net heat residual at each nodal location and over a specified time period. This method will not handle temperature measurement sparsity but was selected as indicated above to provide baseline information such as the problems associated with ill-conditioned equations. Round-off errors and the accuracy of temperatures and temperature derivatives are extremely important considerations.

It should be noted that in this study of the five- and twenty-node models that computer solutions will be used in lieu of experimental data; these computer solutions are as near-perfect temperature data as one can expect. As a result, error effects such as temperature derivative inaccuracies as well as methods comparison should be discernible.

4.1.1 Steady State Conditions

Both the five- and twenty-node models were examined by the use of the steady state least squares equation, which for steady state conditions are deterministic. Under steady state conditions, the number of soft parameters cannot exceed the number of nodes. Various combinations of soft

parameters were studied; the results are shown in Tables 4-1 through 4-5. These tables also include transient results for comparison purposes as well as for convenience.

4.1.1.1 Five-Node Model

Table 4-1 shows the results when three parameters, a_{12}, b_{34}, and b_{56} were considered to be soft and identifiable. This set of parameters is identified as one soft parameter per node. Three sets of soft parameter were examined. These results indicate that the variational method will yield the correct soft parameter values for the case of one soft parameter per node.

The steady state least squares method was studied further by examining the case of two parameters per node with a maximum of five soft parameters, a_{23}, a_{45}, b_{12}, b_{15}, and b_{34}. Again three sets of soft parameters were used as indicated in Table 4-2. These results show that the least squares method lead to inaccurate soft parameter values. A detailed matrix inversion examination revealed that this combination of soft parameters and values yielded and ill-conditioned set of equations. Part of the problem stems from the fact that the values of conduction parameters (conductance), a_{ij}, are an order of magnitude larger than the radiation parameters, $b_{ij}(T_i+T_j)(T_i^2+T_j^2)$, but it is not clear why this set of conditions are ill-conditioned.

In spite of the problems with the case of two parameters per node, it was decided to examine the case of more than two parameters per node. Five parameters, a_{12}, a_{14}, a_{15}, b_{13}, and b_{16}, from a single node were then examined. These results are shown in Table 4-3. Remarkably, very accurate parameter values were obtained. Since the conductors as soft parameters were expected to present the most problems, it was not surprising to find that when the heat inputs, Q_1, Q_2, and Q_3, were considered to be the soft parameters, the calculated heat inputs were extremely accurate as shown in Table 4-4.

In all of the cases discussed above, the soft parameters were identifiable. Suppose that a soft parameter is not included among those selected as a soft parameters or suppose that a hard parameter

TABLE 4-1
FIVE-NODE MODEL, ONE SOFT PARAMETER/NODE
LEAST SQUARES METHOD, STEADY STATE & TRANSIENT

Soft Parameters	PARAMETER VALUES								
	Exact	S.S. Calculated	Trans. Calculated	Exact	S. S. Calculated	Trans. Calculated	Exact	S. S. Calculated	Trans. Calculated
a_{12}	.5	.5	.4682	.75	.74999	.71957	1.0	.999993	.96359
a_{34}	.5	.5	.4999	.25	.25	.25002	1.0	.999993	1.00002
b_{56}	1.2	1.2	1.1985	.6	.6	---	2.4	2.4000	2.39691
b_{56}	---	---	---	1.8	---	1.79773	---	---	---

TABLE 4-2
FIVE-NODE MODEL, TWO SOFT PARAMETERS/NODE
LEAST SQUARES METHOD, STEADY STATE & TRANSIENT

Soft Parameters	PARAMETER VALUES								
	Exact	S. S. Calculated	Trans. Calculated	Exact	S. S. Calculated	Trans. Calculated	Exact	S. S. Calculated	Trans. Calculated
a_{23}	.5	1.34	.4990	.25	.0586	.2517	1.0	1.125	.9910
a_{45}	.5	-1.31	.500	.75	1.320	1.320	1.0	1.125	.9926
b_{12}	.2	-.81	.1600	.3	.5742	.2637	.4	.125	.3304
b_{15}	.2	1.94	.2011	.3	-.2500	.2963	.4	.5	.4106
b_{34}	.2	-2.75	.2009	.1	.7031	.0965	.4	0	.4114

is among those selected as soft parameters, this neglection may have a large impact on the accuracy of the corrected parameter values. For the latter case, as one would expect, the inclusion of a hard parameter among those selected as soft parameters have little effect on the accuracy of the corrected parameter values as shown in Table 4-5. On the other hand, if a soft parameter is not included among those which are considered soft, the impact of the accuracy of the corrected parameter values depends upon the temperature influence of the neglected soft parameter, larger is the influence, larger is the inaccuracy.

The steady state least squares approach using the first node model has revealed severe problems that are directly related to the accuracy of the experimental data. Even with near perfect "measurements" (digital computer output), ill-conditioned situations arose. Another problem of interest is that parallel conductors, $a_{k\ell}$ and $b_{k\ell}$ can not be estimated simultaneously for the steady state case; the reasons for this are apparent from the heat balance equations.

4.1.1.2 Twenty-Node Model

The twenty-node model was studied to evaluate the effect of the model size on the accuracy of the variational approach. Model size is an important consideration since numerical round-off errors increase with an increase in the computation. This is a particularly important problem in the present study because of the large model size objective of the present study coupled with the sophisticated computational schemes used in the correlation methods. The results for the twenty-node model are presented in Table 4-6.

Table 4-6 shows four sets of parameter extimation obtained from the steady state least squares method and one set of parameter estimation from the transient least squares method which will be discussed in a subsequent paragraph. Comparison of the parameter values from the steady state method shows a considerable difference. This difference is due to the manner in which the coefficient matrix was inverted. The coefficient matrix is formulated directly from the normal equations of the steady state least squares method.

TABLE 4-3
FIVE-NODE MODEL, FIVE SOFT PARAMETERS/NODE
LEAST SQUARES METHOD, STEADY STATE & TRANSIENT

Soft Parameters	PARAMETER VALUES								
	Exact	S. S. Calculated	Trans. Calculated	Exact	S. S. Calculated	Trans. Calculated	Exact	S. S. Calculated	Trans. Calculated
a_{12}	.5	.50000	.465	.75	.75000	.745	1	1.00000	.9971
a_{14}	.5	.500002	.515	.25	.25000	.2565	1	1.00002	1.006
a_{15}	.5	.500001	.5002	.75	.75000	.748	1	1.00000	.985
b_{13}	.2	.200001	.199	.3	.30000	.298	.4	.39999	.404
b_{16}	1.2	1.200000	1.1997	1.8	1.8000	1.800	2.4	2.40001	2.400

TABLE 4-4
FIVE-NODE MODEL, HEAT INPUT AS SOFT PARAMETERS
LEAST SQUARES METHOD, STEADY STATE

Soft Parameters	PARAMETER VALUES							
	0% *		10% *		50%*		100%*	
	Exact	Calc.	Exact	Calc.	Exact	Calc.	Exact	Calc.
Q_1	100	100	110	110	150	150	200	200
Q_2	150	150	135	135	750	750	300	300
Q_3	300	300	270	270	450	450	600	600

* Soft Parameters Perturbed 0, 10, 50, & 100%

TABLE 4-5
FIVE-NODE MODEL, SOFT PARAMETERS NOT COMPLETELY IDENTIFIED
LEAST SQUARES METHOD, STEADY STATE

Soft Parameters	Corrected Parameters	Exact	*	Soft Parameters	Corrected Parameters	Exact	*
a_{12}	a_{12}	.5	.5	a_{12}	a_{12}	.5	.5
a_{34}	a_{34}	.5	.500002	a_{34}	a_{34}	.5	.500002
b_{56}	b_{46}	.2	.199999	b_{56}	b_{56}	.2	.199999
					b_{46}	1.2	1.20000

* Nominal values of soft parameters

TABLE 4-6
TWENTY-NODE MODEL, TWENTY LINEAR CONDUCTORS
LEAST SQUARES METHOD, STEADY STATE & TRANSIENT

Parameter	Nominal	Steady State				Transient
		Single Precision			Double Precision	Double Precision
		*	**	***	*	
a_{12}	.5	.499999	.499999	.499998	.499998	.499321
a_{14}		.380354	.505408	-1.06820	-6.37732	.500546
a_{16}		1.15576	.904227	4.50151	38.1957	.500597
$a_{1,17}$		.500003	.500003	.500004	.500003	.499564
$a_{2,3}$		.500000	.500000	.500000	.500000	.500171
$a_{4,5}$		.473159	.446716	-1.28753	1.04286	.500186
$a_{5,6}$		.271532	.471268	-13.1753	-12.6327	.498747
$a_{6,7}$		.543303	.386402	3.05642	2.98912	.500887
$a_{7,8}$		.0472268	.633657	-34.5789	-25.5261	.503956
$a_{8,9}$		.525904	.490942	2.61008	1.98901	.501065
$a_{9,10}$		.652435	.219148	14.7630	9.26232	.474551
$a_{10,11}$		.614255	.600406	8.07551	7.06766	.503780
$a_{11,12}$		.374261	.525606	-8.55224	-6.72789	.500886
$a_{12,13}$		.421169	.626267	-4.95627	-4.03149	.499970
$a_{13,14}$		.627346	.579139	10.0784	7.82042	.499504
$a_{14,15}$		.297302	.510128	-11.1345	-11.1520	.500655
$a_{15,16}$		.383930	.481393	-11.4320	-6.17218	.496552
$a_{17,20}$		.500006	.500006	.500008	.500007	.502554
$a_{18,19}$		.499997	.499997	.499997	.499997	.500282
$a_{19,20}$	.5	.499996	.499996	.499995	.499996	.499731

* Subroutine SØLVIT

** Subroutine INVRSE

*** Subroutine GJR

The three methods of matrix inversion were examined in an attempt to improve the accuracy of the corrected parameters; the methods are identified as subroutines, SØLVIT*, INVRSE, and GJR. Subroutine SØLVIT employs Jordan reduction on the augmented matrix (single and double precision). Subroutine INVERSE is a CINDA-3G subroutine which employs Jordan Reduction with row interchange in single precision. Subroutine GJR is a UNIVAC MATH-PACK subroutine using Gauss-Jordan Reduction with row and column interchanged in single precision.

All three methods in single precision yielded approximately the same results when the parameter values were close to the true value. However, wide variances were found for the remaining parameters. A disturbing result was that SØLVIT in double precision yielded values that were considerably less accurate than SØLVIT in single precision. The details of these matrix inversions are discussed in Appendix D.

4.1.2 Transient Conditions

When the number of soft parameters is greater than the number of nodes or if steady state temperatures are not available, transient conditions must be employed. One of the problems which transient data is the need for accurate temperature derivatives. In the presentation to follow, the importance of accurate temperature derivatives and the application of the least squares by use of transient information on the five- and twenty-node models are discussed.

4.1.2.1 Temperature Derivatives

For transient problems, the temperature derivatives become an important consideration for the accuracy of the soft parameter evaluation. Without experimental data, it was again necessary to employ calculated temperatures via computer solutions. Experimental temperature data can be expected to be less accurate. Initially, temperature derivatives were calculated by employing the finite difference technique which consists of taking temperature difference between two adjacent time points divided by the time increment. Both the forward difference, $T_{i+1}-T_i$, and the historical difference, T_i-T_{i-1},

---

* A subroutine generated by L. C. Fink, programmer TRW Systems Group

were used. The soft parameter corrections were, in general, grossly in error with the use of these temperature derivatives. Better results were obtained by using the average of forward difference and the historical difference. Since the accuracy of the soft parameter evaluation is quite sensitive to the temperature derivative, a further study was made by using a least squares third order polynomial fit of the temperature data. As a matter of interest CINDA-3G has a least squares polynomial (fit of order 10) subroutine called LSTSQU[35]. These temperature derivatives generated from the polynomial fit, in general, yielded the best results although under certain circumstances, the use of the less accurate derivatives yielded as well if not better soft parameter values as shown in Table 4-7. The case of seven soft parameters were studied; note that for these sets of soft parameters the average temperature derivative yield the best result. These results do indicate that the temperature derivative strongly affect the soft parameter values. In view of the importance of the temperature derivatives, the variational approach was studied by using the polynomial fit. No attempt was made to determine the impact on the storage requirements; however, it was noted (Table 4-7) that the computational time for the polynomial fit is appreciably longer than for the other derivative methods.

Table 4-1 shows (paragraph 4.1.1) that for the case of one parameter per node, the use of transient temperature yield parameter values which are not as accurate as for the deterministic case but of sufficient accuracy from a thermal prediction standpoint. This result is not surprising since the temperature derivatives present additional computational inaccuracies.

The two parameters per node case which presented considerable evaluation difficulties using the steady state least squares approach was better handled by the transient least squares formulation as shown in Table 4-2. Attempts to improve the soft parameter evaluation by scaling technique and roundoff check led, surprisingly, to less accurate values as shown in Table 4-8. It is not clear at this time, the cause of the inaccuracies. Logically, the round off error check and scaling should have yielded improved soft parameter values. For the five parameter per node case, the transient least squares method yielded reasonable parameter values but not as accurate as the steady state least squares method as shown in Table 4-3.

TABLE 4-7

EFFECT OF TEMPERATURE DERIVATIVE

Parameters	Exact	Single Time-Step Value	Single Time-Step %Error	Average Value	Average %Error	Polynomial Fit Value	Polynomial Fit %Error
Q_1	100	99.322	-.68	100.013	+.013	100.015	+.015
C_2	.2	.20045	+.23	.200414	+.21	.19848	-.76
C_3	.2	.19491	-2.55	.19955	-.23	.205006	+2.5
C_5	.2	.19339	-3.3	.19952	-.24	.19734	-1.33
a_{25}	.5	.54687	+9.37	.50597	+1.19	.48322	-3.38
a_{35}	.5	.63672	+27.34	.51995	+3.99	.291702	-41.82
b_{14}	.2	.12505	-37.5	.20293	+1.47	.20332	+1.66
Run Time		$\cong$ 42 sec		$\cong$ 42 sec		$\cong$ 42 sec	

TABLE 4-8

FIVE-NODE MODEL, TRANSIENT, TWO SOFT PARAMETERS/NODE

	PARAMETER VALUES								
Soft Parameters	Exact	* Calculated	** Calculated	Exact	* Calculated	** Calculated	Exact	* Calculated	** Calculated
a_{23}	.5	.4990	.386	.25	.2517	.127	1.0	.9910	.762
a_{45}	.5	.5000	-.336	.75	.7555	-.400	1.0	.9926	-.663
b_{12}	.2	.1600	.220	.3	.2637	.276	.4	.3304	.521
b_{15}	.2	.2011	.191	.3	.2963	.211	.4	.4106	.337
b_{34}	.2	.2009	.577	.1	.0965	.561	.4	.4114	.118

* No Scaling or Round-Off Check

** With Scaling and Round-Off Check

With steady state temperature data, the number of parameters that may be corrected cannot exceed the number of measured temperatures; whereas with transient data, this limitation does not exist. Two cases with more than five parameters were examined. Some of the parameter values are in error by a significant amount; others are reasonably accurate. The results are shown in Tables 4-9 and 4-10; note that as the exact value of the soft parameters increases, the estimation becomes more accurate. The reason for this is not clear at this time. Since only transient data can be used to determine the capacitance values, the capability of the transinet least squares method to do this was examined; the results as shown in Table 4-11 are excellent.

Although the parameter values as estimated by the transient least squares method may be inaccurate, the important consideration is the resultant temperature-time history. Using the inaccurate parameter values of the case represented by Table 4-9 (eight soft parameters), the transient temperature history of the five-node model was determined. These results are presented in Table 4-12. Comparison of these temperatures with the temperature history with the exact parameter values as shown in Table A-3, indicate that the temperatures are remarkably close.

4.2 REGRESSION ANALYSIS

The applicability of regression analysis (Refer to paragraph 3.1.7) was investigated by applying it to the five-node thermal model. Again calculated transient temperature histories and the rates of temperature change were used as temperature measurements to correct the perturbed thermal model parameters.

By slicing the transient temperature histories into m slices, we obtained m sets of data; these are

$$\dot{T}_{1,k},\ \dot{T}_{2,k},\ldots\ldots,\ \dot{T}_{5,k};\ T_{1,k},\ T_{2,k},\ldots\ldots,\ T_{5,k}$$

$$k = 1,2,\ldots\ldots,m$$

TABLE 4-9

FIVE-NODE MODEL, TRANSIENT, SEVEN SOFT PARAMETERS

Soft Parameters	Exact	Calculated	Exact	Calculated	Exact	Calculated
Q_1	100	99.9941	100	.99918	100	.99994
C_2	.2	.19528	.3	.29409	.4	.39132
C_3	.2	.20895	.3	.30766	.4	.40785
C_5	.2	.19833	.3	.30129	.4	.40469
a_{25}	.5	.44569	.75	.69142	1.0	.92269
a_{35}	.5	.13325	.75	.56507	1.0	.88021
b_{14}	.2	.203071	.3	.30215	.4	.40168

TABLE 4-10

FIVE-NODE MODEL, TRANSIENT, EIGHT SOFT PARAMETERS

Soft Parameters	Exact	Calculated	Exact	Calculated	Exact	Calculated
a_{12}	.5	.57698	.75	.89954	1.0	.83593
a_{23}	.5	.581717	.25	.32383	1.0	1.2346
a_{25}	.5	.32981	.75	.54542	1.0	.73412
b_{12}	.2	.53592	.3	.06029	.4	.62304
b_{23}	.2	.086176	.3	.19411	.4	.56159
b_{24}	.2	.15796	.3	.10796	.4	.39346
b_{25}	.2	.43611	.3	.58921	.4	.79325
b_{26}	.2	.20133	.3	.29772	.4	.39776

TABLE 4-11

FIVE-NODE MODEL, TRANSIENT, CAPACITANCES AS SOFT PARAMETERS

Soft Parameters	Exact	Calculated	Exact	Calculated	Exact	Calculated
C_1	.2	.19985	.3	.29985	.4	.39995
C_2	.2	.20017	.3	.30010	.4	.40078
C_3	.2	.19980	.3	.29986	.4	.39995
C_4	.2	.19986	.3	.29996	.4	.40005
C_5	.2	.20065	.3	.30080	.4	.40073

TABLE 4-12

FIVE-NODE MODEL, TEMPERATURE RESPONSE WITH INEXACT PARAMETER VALUES*

Time (Hr)	Nodes				
	1	2	3	4	5
0	64.9000	114.600	36.4000	62.1000	102.600
1.0	64.3541	107.669	35.9134	61.6967	89.0159
2.0	63.2498	101.068	34.8341	60.7985	77.7783
3.0	61.7427	94.8059	33.3181	59.5130	68.3247
4.0	59.9482	88.8751	31.4825	57.9239	60.2488
5.0	57.9521	83.2606	29.4161	56.0972	53.2513
6.0	55.8187	77.9433	27.1865	54.0860	47.1082
7.0	53.5963	72.9030	24.8449	51.9329	41.6493
8.0	51.3210	68.1195	22.4310	49.6727	36.7440
9.0	49.0198	63.5734	19.9747	47.3334	32.2912
10.0	46.7131	59.2466	17.4990	44.9384	28.2118
11.0	44.4159	55.1222	15.0216	42.5064	24.4435
12.0	42.1394	51.1846	12.5557	40.0530	20.9368
13.0	39.8917	47.4199	10.1112	37.5908	17.6522
14.0	37.6786	43.8151	7.69583	35.1302	14.5579
15.0	35.5041	40.3586	5.31495	32.6796	11.6284
16.0	33.3711	37.0398	2.97257	30.2460	8.84264
17.0	31.2813	33.8491	6.71505	27.8348	6.18356
18.0	29.2356	30.7778	-1.58638	25.4505	3.63697
19.0	27.2345	27.8181	-3.79994	23.0967	1.19119
20.0	25.2779	24.9627	-5.96857	20.7760	-1.16358
21.0	23.3654	22.2052	-8.09211	18.4907	-3.43558
22.0	21.4965	19.5394	-10.1708	16.2423	-5.63177
23.0	19.6704	16.9602	-12.2049	14.0320	-7.75808
24.0	17.8862	14.4624	-14.1950	11.8606	-9.81965
25.0	16.1428	12.0415	-16.1420	9.72867	-11.8209
26.0	14.4393	9.69347	-18.0466	7.63643	-13.7656
27.0	12.7747	7.41433	-19.9097	5.5839	-15.6571
28.0	11.1477	5.20060	-21.7324	3.57099	-17.4985
29.0	9.55744	3.04908	-23.5155	1.59741	-19.2924
30.0	8.00275	.95607	-25.2601	-.337212	-21.0412

* Inexact parameter values taken from Table 4-10

The five heat balance equations governing this five node model at any time k, when the data are obtained, are given by the following set of equations.

$$(C_i+\Delta C_i)\,\dot{T}_{i,k} = (Q_i+\Delta Q_i) + \sum_{\substack{j=1\\ j\neq i}}^{5} (a_{ij}+\Delta a_{ij})(T_{j,k}-T_{i,k}) + \sum_{\substack{j=1\\ j\neq 1}}^{5} (b_{ij}+\Delta b_{ij})(T^4_{j,k}-T^4_{k,k}) \tag{4-1}$$

$$i = 1,2,3,4,5$$

Where the C_i, Q_i, a_{ij}, and b_{ij} are nominal parameter values and the Δ's are the necessary corrections to these values such that a best fit to the m sets of data can be realized. If the parameters are hard (the nominal values correspond to the true values), the Δ's associated with these parameters are set to zero. To obtain a best fit to the m sets of data, we wish to determine the Δ's in the least squares sense by minimizing the following criterion functions.

$$\varepsilon_i = \sum_{k=1}^{m} \{(C_i+\Delta C_i)\,\dot{T}_{i,k} - (Q_i+\Delta Q_i) - \sum_{\substack{j=1\\ j\neq i}}^{5} (a_{ij}+\Delta a_{ij})(T_{j,k}-T_{i,k}) -$$

$$\sum_{\substack{j=1\\ j\neq i}}^{5} (b_{ij}+\Delta b_{ij})(T^4_{j,k}-T^4_{i,k})\}^2 \tag{4-2}$$

$$i = 1,2,3,4,5$$

Upon differentiating ε_i with respect to the Δ's and setting each of the resultant equation to zero, we get the following sets of normal equations,

$$\sum_{k=1}^{m} \dot{T}_{i,k} J_{i,k} = 0 \tag{4-3}$$

$$\sum_{k=1}^{m} - J_{i,k} = 0 \tag{4-4}$$

$$\sum_{k=1}^{m} - (T_{j,k}-T_{i,k})\, J_{i,k} = 0 \; ; \; j = 1,2,\ldots,5 \tag{4-5}$$

$$\sum_{k=1}^{m} - (T^4_{j,k}-T^4_{i,k})\, J_{i,k} = 0 \; ; \; j = 1,2,\ldots,5 \tag{4-6}$$

where

$$J_{i,k} = \{(C_i+\Delta C_i)\, \dot{T}_{i,k} - (Q_i+\Delta Q_i) - \sum_{\substack{j=1 \\ j\neq i}}^{5} (a_{ij}+\Delta a_{ij})(T_{j,k}-T_{i,k}) -$$

$$\sum_{\substack{j=1 \\ j\neq i}}^{5} (b_{ij}+\Delta b_{ij})(T^4_{j,k}-T^4_{i,k})\} \tag{4-7}$$

$$i = 1,2,\ldots,5$$

By solving for the Δ's in the first set of normal equations corresponded to i = 1, the corrections to the various soft parameters associated with node 1 are obtained. These corrections are then substituted into the second set of normal equations corresponding to i = 2, wherever they occurred; this reduces the number of unknown Δ's and consequently the number of equations in the second set of normal equations. The remaining Δ's in the second set of normal equations are then found. Upon having determined the corrections to the various soft thermal model parameters associated with node 2, they are substituted into the third set of normal equations corresponding to node 3 together with the corrections determined for node 1, wherever they occur; this reduces the number of unknown Δ's and consequently the number of equations in the third set of normal equations by a greater amount. The remaining Δ's in the third set of normal equations are then found. This sequential scheme continues until all the Δ's are solved for (until all five sets of normal equations are considered).

The above equations were programmed and a number of test cases examined. These results were found to be quite unsatisfactory. It appeared that the probable cause of the inaccurate estimate was due

to inaccurate temperature derivative. Because of adequate baseline information obtained from the least squares method, no further attempt was made to continue the regression analysis method.

4.3 KALMAN FILTERING METHOD

The Kalman filtering method was discussed in considerable detail in paragraph 3.2.1. The sequential aspects of this method were examined by using the five- and twenty-node models and a number of test cases. Since these test cases employed numerically calculated temperature histories, which are near perfect, it was necessary to construct an artificial filtering problem.

The formulation of this artificial filtering problem is as follows. The governing heat balance equations are first arranged in the following form,

$$\{y^*\}=[M]\{x\} + \{W\} \tag{4-8}$$

This is done by first placing those terms that involve the known (hard) model parameters on the left hand side; these left hand terms are combined to form an artificial measurement vector $\{y^*\}$. The remaining terms that involve the unknown (soft) model parameters are placed on the right hand side to form $[M]\{x\}$, where $\{x\}$ is the state vector formed by the unknown model parameters, and $[M]$ is the artificial measurement matrix that involve the coefficients of the unknown parameters. Since all the unknown parameters are considered to be constant, the following relation holds,

$$\{x\}_{t+\Delta t} = \{x\}_t \tag{4-9}$$

This gives an identity matrix for our updating matrix $[U]$. With the vectors $\{x\}$, $\{y^*\}$, and the matrices $[M]$ and $[U]$ formulated, the Kalman filter equations can be used to correct the thermal model parameters sequentially as soon as the problem is initialized by assuming a priori knowledge of the unknown parameters, i.e., the assumptions of $\{x_a\}$ and $[A]$ (refer to paragraph 3.2.1).

In perturbing the test models, the five-node model was first considered with the measurement noise vector $\{W\}$ set to zero; this means that the Kalman filtering scheme is being used to solve a deterministic problem. The first case tried was the correction of

five soft parameters, two linear and three radiation; this case is the same as the one presented in Table 4-2 with the exception that the Stefan-Boltzmann constant, σ, was "rounded" to 2.0×10^{-9} BTU/hr-ft^2-°R^4. The results as presented in Table 4-13 were quite unsatisfactory; a convergence trend was indicated after two sets of data were processed but divergence from the true values occurred when additional data were processed.

A detailed examination of the computer printouts revealed that the state error covariance matrix, which should be a symmetric and positive semidefinite matrix, became asymmetric after having processed the first set of data; the asymmetry became more and more pronounced when additional data were processed. The asymmetry of the state error covariance matrix caused the parameters to diverge from the true values. The cause of this asymmetry appeared to be numerical round-off errors even though specific sources could not be isolated. As a result a new subroutine called SYMFRC was generated to force symmetry of the off-diagonal elements by averaging the values of the elements. SYMFRC was employed to force the symmetry in the state covariance matrix. With this forced symmetircal matrix, the case with the two linear and three radiation parameters was reexamined. The results as shown in Table 4-14 were excellent.

The Kalman filter was examined further by studing the following cases: (1) four linear and four radiation conductors with no parallel sets; (2) one capacitor, three linear, and four radiation conductors with no parallel set; (3) four linear and four radiation conductors with one parallel set; (4) four linear and four radiation conductors with all sets parallel; and (5) three linear and five radiation conductors with three parallel sets. The results are tabulated in Table 4-15 to 4-19.

It can be observed that in all cases with the exception of case (5), (Table 4-19), excellent correlation between the calculated parameter values and their true values were obtained. The parameter values converged closer and closer to their true values as additional temperature data were processed with the exception of case (4), (Table 4-18). Case (4) (Table 4-18) shows an undesirable oscillation in one parallel pair of parameters. Case (5) shows that all the perturbed parameters converged

TABLE 4-13
FIVE-NODE MODEL, KALMAN FILTER
2 Linear and 3 Radiation Conductors
(No Parallel Set, No Noise)

Unknown Parameters	True Values	A Priori Estimates	Estimates After 1st Set of Data	Estimates After 2nd Set of Data	Estimates After 3rd Set of Data	Estimates After 4th Set of Data
a_{23}	0.5	0.2	0.461329	0.489715	0.510793	1.24624
a_{45}	0.5	0.8	0.406523	0.487679	0.514954	1.64470
σb_{12}	4.0×10^{-10}	9.0×10^{-10}	1.38667×10^{-9}	6.30244×10^{-10}	7.37685×10^{-10}	-1.13190×10^{-7}
σb_{15}	4.0×10^{-10}	1.0×10^{-10}	-8.02345×10^{-12}	4.50841×10^{-10}	3.72394×10^{-10}	5.32518×10^{-9}
σb_{34}	4.0×10^{-10}	7.0×10^{-10}	6.89795×10^{-10}	6.20965×10^{-10}	4.41010×10^{-10}	2.82377×10^{-3}

TABLE 4-14
FIVE-NODE MODEL, KALMAN FILTER
2 Linear and 3 Radiation Conductors
(No Parallel Set, No Noise, Forced Symmetry of State Error Covariance Matrix)

Unknown Parameters	True Values	A Priori Estimates	Estimates After 1st Set of Data	Estimates After 2nd Set of Data	Estimates After 3rd Set of Data	Estimates After 4th Set of Data
a_{23}	0.5	0.2	0.570977	0.501667	0.500471	0.500226
a_{45}	0.5	0.8	0.770503	0.510508	0.500267	0.500122
σb_{12}	4.0×10^{-10}	9.0×10^{-10}	6.20007×10^{-10}	4.17828×10^{-10}	4.00339×10^{-10}	4.00124×10^{-10}
σb_{15}	4.0×10^{-10}	1.0×10^{-10}	-6.33280×10^{-10}	4.39058×10^{-10}	4.00065×10^{-10}	3.99878×10^{-10}
σb_{34}	4.0×10^{-10}	7.0×10^{-10}	7.17983×10^{-10}	3.98296×10^{-10}	4.00041×10^{-10}	3.99991×10^{-10}

TABLE 4-15

FIVE-NODE MODEL, KALMAN FILTER
4 Linear and 4 Radiation Conductors
(No Parallel Set, No Noise, Forced Symmetry of State Error Covariance Matrix)

Unknown Parameters	True Values	A Priori Estimates	Estimates After 1st Set of Data
a_{12}	0.5	0.9	0.677483
a_{23}	0.5	0.1	0.353787
a_{34}	0.5	0.7	0.524037
a_{45}	0.5	0.3	0.356589
σb_{13}	4.0×10^{-10}	7.0×10^{-10}	7.91712×10^{-10}
σb_{24}	4.0×10^{-10}	1.0×10^{-10}	5.32499×10^{-10}
σb_{35}	4.0×10^{-10}	6.0×10^{-10}	5.07647×10^{-10}
σb_{46}	4.0×10^{-10}	2.0×10^{-10}	3.99998×10^{-10}

Estimates After 2nd Set of Data	Estimates After 3rd Set of Data	Estimates After 4th Set of Data	Estimates After 5th Set of Data
0.501084	0.498899	0.499385	0.499749
0.500312	0.500215	0.500102	0.500014
0.499667	0.500274	0.500068	0.500054
0.499084	0.500118	0.500051	0.500003
4.04943×10^{-10}	3.98845×10^{-10}	3.99541×10^{-10}	3.99863×10^{-10}
3.99971×10^{-10}	4.00678×10^{-10}	4.00325×10^{-10}	4.00238×10^{-10}
4.01159×10^{-10}	4.00076×10^{-10}	4.00146×10^{-10}	4.00006×10^{-10}
3.99998×10^{-10}	3.99998×10^{-10}	3.99998×10^{-10}	3.99998×10^{-10}

TABLE 4-16

FIVE-NODE MODEL, KALMAN FILTER
1 Capacitance, 3 Linear and 4 Radiation Conductors
(No Parallel Set, No Noise, Forced Symmetry of State Error Covariance Matrix)

Unknown Parameters	True Values	A Priori Estimates	Estimates After 1st Set of Data
C_1	0.2	0.1	0.120680
a_{23}	0.5	0.1	0.380758
a_{34}	0.5	0.7	0.655369
a_{45}	0.5	0.3	0.296481
σb_{12}	4.0×10^{-10}	7.0×10^{-10}	9.07138×10^{-10}
σb_{24}	4.0×10^{-10}	1.0×10^{-10}	2.55790×10^{-10}
σb_{35}	4.0×10^{-10}	6.0×10^{-10}	5.52767×10^{-10}
σb_{46}	4.0×10^{-10}	2.0×10^{-10}	2.59235×10^{-10}

Estimates After 2nd Set of Data	Estimates After 3rd Set of Data	Estimates After 4th Set of Data	Estimates After 5th Set of Data
0.183035	0.200406	0.199996	0.200000
0.463055	0.490780	0.499836	0.500000
0.478851	0.516216	0.500509	0.500002
0.449538	0.498579	0.500038	0.500000
4.83019×10^{-10}	3.88368×10^{-10}	3.99807×10^{-10}	3.99998×10^{-10}
$4,42322\times10^{-10}$	4.49507×10^{-10}	4.01858×10^{-10}	3.99997×10^{-10}
$4,16606\times10^{-10}$	3.98055×10^{-10}	3.99932×10^{-10}	3.99994×10^{-10}
4.04415×10^{-10}	4.01383×10^{-10}	4.00016×10^{-10}	3.99999×10^{-10}

TABLE 4-17

FIVE-NODE MODEL, KALMAN FILTER
4 Linear and 4 Radiation Conductors
(One Parallel Set, No Noise, Forced Symmetry of State Error Covariance Matrix)

Unknown Parameters	True Values	A Priori Estimates	Estimates After 1st Set of Data
a_{12}	0.5	0.9	0.418620
a_{23}	0.5	0.1	0.446879
a_{34}	0.5	0.7	0.529639
a_{45}	0.5	0.3	0.354025
σb_{12}	4.0×10^{-10}	7.0×10^{-10}	5.28459×10^{-10}
σb_{24}	4.0×10^{-10}	1.0×10^{-10}	5.40279×10^{-10}
σb_{35}	4.0×10^{-10}	6.0×10^{-10}	5.09572×10^{-10}
σb_{46}	4.0×10^{-10}	2.0×10^{-10}	3.99998×10^{-10}

Estimates After 2nd Set of Data	Estimates After 3rd Set of Data	Estimates After 4th Set of Data	Estimates After 5th Set of Data
0.499195	0.500007	0.500014	0.500026
0.500781	0.500017	0.500001	0.500000
0.497533	0.499982	0.500000	0.500002
0.499224	0.500037	0.500004	0.500002
4.01271×10^{-10}	3.99990×10^{-10}	3.99978×10^{-10}	3.99956×10^{-10}
3.98061×10^{-10}	3.99923×10^{-10}	3.99988×10^{-10}	3.99987×10^{-10}
4.00503×10^{-10}	3.99997×10^{-10}	4.00001×10^{-10}	3.99995×10^{-10}
3.99998×10^{-10}	3.99998×10^{-10}	3.99998×10^{-10}	3.99998×10^{-10}

TABLE 4-18
FIVE-NODE MODEL, KALMAN FILTER
4 Linear and 4 Radiation Conductors
(Four Parallel Sets, No Noise, Forced Symmetry of State Error Covariance Matrix)

Unknown Parameters	True Values	A Priori Estimates	Estimates After 1st Set of Data	Estimates After 2nd Set of Data
a_{12}	0.5	0.9	0.423799	0.422071
a_{23}	0.5	0.1	0.443865	0.443865
a_{34}	0.5	0.7	0.456180	0.459790
a_{45}	0.5	0.3	0.535847	0.518807
σb_{12}	4.0×10^{-10}	7.0×10^{-10}	5.27431×10^{-10}	5.28538×10^{-10}
σb_{23}	4.0×10^{-10}	1.0×10^{-10}	2.03844×10^{-10}	2.03844×10^{-10}
σb_{34}	4.0×10^{-10}	6.0×10^{-10}	4.73948×10^{-10}	4.77567×10^{-10}
σb_{45}	4.0×10^{-10}	2.0×10^{-10}	3.39281×10^{-10}	3.66378×10^{-10}

Estimates After 3rd Set of Data	Estimates After 4th Set of Data	Estimates After 5th Set of Data	Estimates After 6th Set of Data
0.484212	0.498410	0.498702	0.500680
0.523878	0.619702	0.527821	0.788397
0.495009	0.500655	0.499347	0.501029
0.501422	0.499883	0.500271	0.499602
4.27578×10^{-10}	4.02987×10^{-10}	4.02524×10^{-10}	3.98752×10^{-10}
2.25517×10^{-10}	2.40879×10^{-10}	1.36164×10^{-10}	-1.66470×10^{-10}
4.10003×10^{-10}	3.98738×10^{-10}	4.01442×10^{-10}	3.97814×10^{-10}
3.97416×10^{-10}	4.00166×10^{-10}	3.99396×10^{-10}	4.00786×10^{-10}

TABLE 4-19

FIVE-NODE MODEL, KALMAN FILTER

3 Linear and 5 Radiation Conductors

(3 Parallel Sets, No Noise, Forced Symmetry of State Error Covariance Matrix)

Unknown Parameters	True Values	A Priori Estimates	Estimates After 1st Set of Data	Estimates After 2nd Set of Data	Estimates After 3rd Set of Data	Estimates After 4th Set of Data
a_{12}	0.5	0.9	0.619710	0.500010	0.496027	-37.3930
a_{23}	0.5	0.1	0.388424	0.499476	0.501868	-12.6194
a_{25}	0.5	0.7	0.593185	0.500007	0.499040	9.44752
σb_{12}	4.0×10^{-10}	7.0×10^{-10}	3.46594×10^{-10}	3.99995×10^{-10}	4.01928×10^{-10}	193.759×10^{-10}
σb_{23}	4.0×10^{-10}	1.0×10^{-10}	4.51504×10^{-10}	4.00248×10^{-10}	3.99058×10^{-10}	72.2106×10^{-10}
σb_{24}	4.0×10^{-10}	1.0×10^{-10}	4.00001×10^{-10}	4.00002×10^{-10}	4.00005×10^{-10}	4.0008×10^{-10}
σb_{25}	4.0×10^{-10}	6.0×10^{-10}	3.58742×10^{-10}	3.99995×10^{-10}	4.00478×10^{-10}	-42.2503×10^{-10}
σb_{26}	4.0×10^{-10}	2.0×10^{-10}	4.00000×10^{-10}	4.00000×10^{-10}	4.00000×10^{-10}	4.00071×10^{-10}

to values within fractions of one percent of their true values after having processed two sets of temperature data, and then diverged to undesirable results.

Detailed examination of the Kalman filter equations indicated that numerical errors probably caused the divergence and oscillations. The nature of the Kalman filter equations is such that by setting the measurement noise vector to zero, it is necessary to invert a matrix given by $[M][A][M]^T$ (refer to Equation (3-81)). When the unknown parameter values approaches their true values, the elements in the state error covariance matrix becomes very small. The inversion of matrices whose elements are very small poses numerical problem. This problem was eliminated by employing an artificial measurement noise vector whose elements are small, stationary and uncorrelated; case (5) was then rerun. The results are tabulated in Table 4-20. It can be observed that excellent correlations between the calculated values and their true values were obtained; convergence to the true parameter values were obtained as more and more temperature data were processed.

For the twenty-node model, a case with twenty linear conductors perturbed. Excellent results were obtained as tabulated in Table 4-21. A further study was made by perturbing all the parameters with the exception of the heat input, Q's. The number of soft parameters totaled eighty-four (84) as shown in Table A-5. The results were excellent as shown in Table 4-22. As a note of interest, this case required approximately 20,000 core locations and a run time of approximately 4 minutes (nine passes).

Another particular note of importance is that no attempt was made to include temperature sparsity because of programming complexities and because of problems confronted with this temperature sparsity consideration using Program MAFIA. These problems are applicable to both correlation techniques; discussion of these problems are found in the next section and Appendix C.

Unperturbed mathematical models were analyzed in order to produce simulated test data. In the early stages the temperatures at two successive time points divided by the time step were used for the temperature derivatives. However, this provided the T dot value over

TABLE 4-20
FIVE-NODE MODEL, KALMAN FILTER
3 Linear and 5 Radiation Conductors
With Measurement Noise
(3 Parallel Sets, Artificial Noise, Forced Symmetry of State Error Covariance Matrix)

Unknown Parameters	True Values	A Priori Estimates	Estimates After 1st Set of Data	Estimates After 2nd Set of Data	Estimates After 3rd Set of Data	Estimates After 4th Set of Data
a_{12}	0.5	0.9	0.418623	0.497095	0.499031	0.499395
a_{23}	0.5	0.1	0.582553	0.501575	0.500374	0.500194
a_{25}	0.5	0.7	0.468502	0.498860	0.499763	0.499869
σb_{12}	4.0×10^{-10}	7.0×10^{-10}	5.28460×10^{-10}	4.04655×10^{-10}	4.01568×10^{-10}	4.00986×10^{-10}
σb_{23}	4.0×10^{-10}	1.0×10^{-10}	2.59599×10^{-10}	3.97270×10^{-10}	3.99339×10^{-10}	3.99651×10^{-10}
σb_{24}	4.0×10^{-10}	1.0×10^{-10}	3.99996×10^{-10}	3.99993×10^{-10}	3.99994×10^{-10}	3.99994×10^{-10}
σb_{25}	4.0×10^{-10}	6.0×10^{-10}	4.48009×10^{-10}	4.01848×10^{-10}	4.00416×10^{-10}	4.00242×10^{-10}
σb_{26}	4.0×10^{-10}	2.0×10^{-10}	3.99994×10^{-10}	3.99997×10^{-10}	3.99999×10^{-10}	3.99999×10^{-10}

Estimates After 5th Set of Data	Estimates After 6th Set of Data	Estimates After 7th Set of Data	Estimates After 8th Set of Data
0.499524	0.499571	0.499649	0.499782
0.500125	0.500084	0.500052	0.500031
0.499911	0.499934	0.499942	0.499950
4.00777×10^{-10}	4.00701×10^{-10}	4.00575×10^{-10}	4.00360×10^{-10}
3.99772×10^{-10}	3.99844×10^{-10}	3.99903×10^{-10}	3.99941×10^{-10}
3.99994×10^{-10}	3.99994×10^{-10}	3.99994×10^{-10}	3.99995×10^{-10}
4.00170×10^{-10}	4.00132×10^{-10}	4.00118×10^{-10}	4.00106×10^{-10}
4.00000×10^{-10}	4.00000×10^{-10}	3.99999×10^{-10}	3.99998×10^{-10}

TABLE 4-21
TWENTY-NODE MODEL, KALMAN FILTER
(Twenty Linear Conductors, Artificial Noise, Forced Symmetry of State Error Covariance Matrix)

Unknown Parameters	True Values	A Priori Estimates	Estimates After 1st Set of Data	Estimates After 2nd Set of Data	Estimates After 4th Set of Data	Estimates After 5th Set of Data
$a_{1,2}$	0.5	0.1	0.499799	0.500002	0.500012	0.499995
$a_{1,11}$	0.5	0.2	0.498765	0.500002	0.500120	0.499961
$a_{1,16}$	0.5	0.3	0.501530	0.499999	0.499928	0.500024
$a_{1,13}$	0.5	0.4	0.499997	0.499994	0.499992	0.500000
$a_{2,3}$	0.5	0.9	0.499704	0.499998	0.499990	0.500004
$a_{4,5}$	0.5	0.8	0.497125	0.500064	0.500366	0.499878
$a_{5,6}$	0.5	0.7	0.502372	0.499990	0.499895	0.500046
$a_{6,7}$	0.5	0.6	0.498517	0.499999	0.499997	0.499995
$a_{7,8}$	0.5	0.1	0.492275	0.500096	0.500964	0.499626
$a_{8,9}$	0.5	0.2	0.506724	0.499902	0.499963	0.499966
$a_{9,10}$	0.5	0.3	0.527706	0.499849	0.499946	0.499941
$a_{10,11}$	0.5	0.4	0.465207	0.499624	0.499765	0.499757
$a_{11,12}$	0.5	0.9	0.461605	0.500138	0.500010	0.500075
$a_{12,13}$	0.5	0.8	0.502977	0.500025	0.499979	0.500029
$a_{13,14}$	0.5	0.7	0.506603	0.499969	0.499273	0.500308
$a_{14,15}$	0.5	0.6	0.501387	0.500006	0.499984	0.500009
$a_{15,16}$	0.5	0.1	0.497524	0.499995	0.500136	0.499955
$a_{17,20}$	0.5	0.2	0.500593	0.499890	0.500006	0.499959
$a_{18,19}$	0.5	0.3	0.316130	0.499295	0.499578	0.499799
$a_{19,20}$	0.5	0.4	0.500283	0.499955	0.499983	0.499996

TABLE 4-22

TWENTY-NODE MODEL, KALMAN FILTER
36 Linear & 28 Radiative Conductors, 20 Capacitances
(Artificial Noise, Forced Symmetry of State Error Covariance Matrix)

Unknown Parameters	True Value	Estimate After 9th Set of Data	Unknown Parameters	True Value	Estimate After 9th Set of Data	Unknown Parameters	True Value	Estimate After 9th Set of Data
C_1	.05	.0498935	a_{45}	.5	.502054	$\sigma b_{1,21}$	*	5.04613×10^{-10}
C_2		.0499921	$a_{4,20}$		.489397	$\sigma b_{2,21}$		4.99538x
C_3		.0500326	a_{56}		.500873	$\sigma b_{3,21}$		5.01477x
C_4		.0496892	a_{58}		.501775	$\sigma b_{4,21}$		5.00575x
C_5		.0501638	$a_{5,20}$		.500681	$\sigma b_{9,21}$		4.97538x
C_6		.0500891	a_{67}		.500894	$\sigma b_{10,21}$		5.00803x
C_7		.0477995	a_{78}		.4568612	$\sigma b_{11,21}$		4.98179x
C_8		.0507621	$a_{7,10}$		.501038	$\sigma b_{12,21}$		4.83844x
C_9		.0497347	a_{89}		.496495	$\sigma b_{17,21}$		4.99464x
C_{10}		.0500998	$a_{8,19}$		.511881	$\sigma b_{20,21}$		5.04621x
C_{11}		.0498834	$a_{9,10}$		.499504	$\sigma b_{7,21}$		4.87098x
C_{12}		.0480263	$a_{9,12}$		.496274	$\sigma b_{15,21}$		4.89399x
C_{13}		.0493641	$a_{9,19}$		.499056	$\sigma b_{18,21}$		5.24572x
C_{14}		.0498457	$a_{10,11}$		.500379	$\sigma b_{19,21}$		4.77062x
C_{15}		.0491943	$a_{11,12}$		.496140	$\sigma b_{8,21}$		5.04009x
C_{16}		.0504122	$a_{11,14}$		.498470	$\sigma b_{16,21}$		5.15093x
C_{17}		.0499071	$a_{12,13}$		.496418	$\sigma b_{7,15}$		3.90605x
C_{18}		.0506099	$a_{12,18}$		.470691	$\sigma b_{8,16}$		5.53624x
C_{19}		.0497443	$a_{13,14}$		.498318	$\sigma b_{7,16}$		4.17923x
C_{20}	.05	.0493960	$a_{13,16}$		.495731	$\sigma b_{8,15}$		4.06697x
a_{12}	.5	.500243	$a_{13,18}$		.496732	$\sigma b_{7,18}$		5.02361x
a_{14}		.497493	$a_{14,15}$		.498458	$\sigma b_{7,19}$		5.34679x
a_{16}		.497666	$a_{15,16}$		.523403	$\sigma b_{8,18}$		5.54961x
a_{17}		.498447	$a_{16,17}$		.500182	$\sigma b_{8,19}$		5.20843x
a_{23}		.500335	$a_{17,18}$		.496115	$\sigma b_{15,18}$		5.95466x
$a_{2,15}$		.500291	$a_{17,20}$		.499229	$\sigma b_{15,19}$		5.10081x
a_{34}		.497182	$a_{18,19}$		.515076	$\sigma b_{16,19}$		4.03536x
a_{36}	.5	.500888	$a_{19,20}$	.5	.465891	$\sigma b_{16,8}$	*	4.83587×10^{-10}

* For Convenience, σb_{ij} were all set equal to 5.0×10^{-10}

a time space when it was required at a time point. Consequently, several parameter correction methods diverged due to this inaccuracy. A subroutine was then coded which utilized the pseudo-compute sequence to calculate the net heat flow into each node at a time point; these values divided by the nodal capacitance provides an accurate time point T dot. While this method is suitable for simulated test data, actual test data will require smoothing and fitting techniques in order to obtain fairly accurate T dot values.

4.4 PROGRAM MAFIA

The correlation method denoted in Program MAFIA was discussed in paragraph 3.1.5. This method was applied to the five-node thermal mdoel with calculated temperature histories as the temperature measurements. These artificial temperature measurements were then used to estimate the unknown (soft) thermal model parameters.

A total of three cases were considered. The first case consisted of estimating ten parameters (five parallel sets) assuming all five nodes measured. The second case consisted of estimating the same ten parameters with an additional one representing the initial temperature of node 4 and assuming only four nodes measured; these measured nodes are 1, 2, 3, and 5. The third case consisted of estimating the same ten parameters with two additional parameters representing the initial temperatures of nodes 2 and 4 and assuming only three nodes measured; these measured nodes ar 1, 3, and 5.

For the first two cases, two perturbations each were considered. These perturbations represented one with a priori knowledge and the other without a priori knowledge of the unknowns. The results for these two cases are tabulated in Tables 4-23 & 4-24. From Table 4-23 it can be observed that relative good correlations between the estimated values and their true values were realized. A comparison of the estimates between those obtained with a priori information concerning the unknowns and those obtained without a priori information showed that better results were obtained without a priori information. This was due to the fact that in the method of least squares, the a priori informations are used as additional measurements. When

TABLE 4-23

FIVE-NODE MODEL, PROGRAM MAFIA
Five Nodes Measured

Unknown Parameters	With a Priori Informations			Without a Priori Informations	True Values
	a Priori Estimates	σ(Weights for the a Priori Estimates)	Calculated Values	Calculated Values	
a_{12}	0.8	0.25	0.449	0.455	0.5
a_{15}	1.0	↑	0.549	0.541	↑
a_{23}	0.3	↑	0.479	0.487	↑
a_{34}	0.1	↑	0.537	0.499	↑
a_{45}	0.4	0.25	0.388	0.508	0.5
b_{12}	0.3	0.1	0.245	0.241	0.2
b_{15}	0.4	↓	0.135	0.144	↓
b_{23}	0.1	↓	0.229	0.218	↓
b_{34}	0.5	↓	0.152	0.202	↓
b_{35}	0.7	0.1	0.348	0.185	0.2

TABLE 4-24

FIVE-NODE MODEL, PROGRAM MAFIA
Four Nodes Measured

Unknown Parameters	With a Priori Informations			Without a Priori Informations	True Values
	a Priori Estimates	σ(Weights for the a Priori Estimates)	Calculated Values	Calculated Values	
$T_4(t_o)$	571.8	50	521.3	521.3	521.8
a_{12}	0.8	0.5	0.467	0.471	0.5
a_{15}	1.0	↕	0.563	0.549	↕
a_{23}	0.3	↕	0.512	0.514	↕
a_{34}	0.1	↕	0.394	0.388	↕
a_{45}	0.4	0.5	0.420	0.445	0.5
b_{12}	0.3	0.5	0.237	0.235	0.2
b_{15}	0.4	↕	0.123	0.141	↕
b_{23}	0.1	↕	0.186	0.183	↕
b_{34}	0.5	↕	0.339	0.348	↕
b_{45}	0.7	0.5	0.286	0.254	0.2

a priori informations are used, proper weights to these a priori informations must also be used. When improper weights are used, the a priori informations becomes a bad set of data, and with this additional bad set of data will give inferior results than estimation without using this bad set of data. This comparison points out the fact that when the method of least squares is to be used, it is often preferable to use it without a priori informations. If a priori informations are to be used, a relatively accurate knowledge of the errors of the a priori estimates must be available, i.e., the knowledge of the error convariance matrix concerning the a priori estimates.

The results of the third case with the same five-node thermal model as used in the first and second case was found to be quite unsatisfactory. These results are tabulated in Table 4-25. Although the results shows that the values obtained for the unknown conduction conductances were in serious error and the initial temperatures estimated for the unmeasured nodes; nodes 2 and 4 were inaccurate by approximately +18 and -18°F, respectively. The reason for these results became evident after a close scrutiny of the thermal model. The heat balance equations governing nodes 2 and 4 were found to be identical. This meant that node 2 and node 4 could not be distinguished, (observed) and the simultaneous estimation of the unknown parameters and initial temperatures at nodes 2 and 4 could not be accomplished.

The problem of observability was studied further by using a slightly modified five-node thermal model constructed by eliminating the couplings between nodes 1 and 4 and nodes 2 and 5 from the original five-node model. The decoupling appeared to satisfy observability between nodes 2 and 4. The theoretically calculated temperature histories for nodes 1, 3 and 5 for this model were then used as measurement data to estimate the ten unknown parameters and the initial temperatures for nodes 2 and 4. The results are tabulated in Table 4-26. It can be observed that correlation between the estimated values and their true values was not satisfactory.

TABLE 4-25
FIVE-NODE MODEL, PROGRAM MAFIA
Three Nodes Measured
(Nodes 2 and 4 Indistinguishable)

Unknown Parameters	a Priori Estimates	σ(Weight for the a Priori Estimates)	Calculated Values	True Values
T_2 (t_o)	567.	50.	585.4	567.3
T_4 (t_o)	522.	50.	503.2	521.8
a_{12}	0.8	0.5	0.441	0.5
a_{15}	1.0	↕	0.578	↕
a_{23}	0.3	↕	0.488	↕
a_{34}	0.1	↕	0.526	↕
a_{45}	0.4	0.5	0.390	0.5
b_{12}	0.3	0.5	0.255	0.2
b_{15}	0.4	↕	0.090	↕
b_{23}	0.1	↕	0.220	↕
b_{34}	0.5	↕	0.161	↕
b_{45}	0.7	0.5	0.363	0.2

TABLE 4-26
FIVE-NODE MODEL, PROGRAM MAFIA
Three Nodes Measured
(Nodes 2 and 4 Assumed to be Distinguishable)

Unknown Parameters	Calculated Values	True Values
T_2 (t_o)	559.6	567.9
T_4 (t_o)	521.5	521.7
a_{12}	-0.247	0.5
a_{15}	0.678	↕
a_{23}	0.382	↕
a_{34}	0.907	↕
a_{45}	0.715	0.5
b_{12}	1.14	0.2
b_{15}	-0.131	↕
b_{23}	0.403	↕
b_{34}	-0.171	↕
b_{45}	0.138	0.2

At the present time, the only plausible explanation is that the observability was not satisfied. A discussion of observability of the thermal networks is presented in Appendix C.

4.5 SUMMARY

The results of the studied correlation methods have revealed a number of interesting problem areas such as temperature sparsity and numerical round-off errors and at the same time have yielded encouraging parameter estimation possibilities. The method of least squares with the net heat flux residual provided considerable insight on the accuracy needs of the correlation methods which do not account for noisy measurements. The Kalman filter revealed that the correction of a large number of parameters is indeed feasible under relatively ideal conditions; it remains to be seen how less-than-ideal conditions affects the parameter estimation qualities of the Kalman filter. Program MAFIA revealed some of the problems of temperature sparsity and directed some preliminary effort in the area of observability of thermal networks.

5.0 PROBLEM AREAS AND SYSTEM PROGRAMMING CONSIDERATIONS

Parameter estimation of a large thermal network presents many technical difficulties, some of which are directly applicable to the correlation methods and the remainder concerned with related and highly important areas. Systems programming represents a critically important consideration that must be resolved before a practical correlation method becomes a reality. The presently recognized and expected problem areas and a few of the planned system programming aspects are discussed.

5.1 PROBLEM AREAS

5.1.1 Sparsity of Temperature Measurements

The accommodation of sparse temperature measurements remains a highly critical and presently unsolved problem. Its solution represents a necessary condition before a meaningful and practical correlation method is possible. This problem has been briefly explored in the present study and is reported in Appendix C. The results are rather incomplete and fragmentary but are presented at this time only to indicate the nature and significance of the general problem of observability of nonlinear systems and its relevance to thermal network correction.

5.1.2 Temperature Dependent Coefficients

From theoretical considerations, temperature dependent coefficients are not expected to present undue difficulties. For example, if the coefficients can be expressed as a product of two terms, one being temperature dependent and the other constant with only the constant part being correctible, the correlation methods are directly applicable. Temperature dependent coefficients were discussed in Paragraph 3.2.1, Kalman Filter. If the temperature dependent coefficients cannot be adequately represented in this manner, an alternative is to assume a constant coefficient over a specified time period; this yields a stepwise representation of the coefficients. Possible problem areas include difficulties that may be encountered because of the increased non-linearity of the heat balance equations and program difficulties because of the need for increased core locations.

5.1.3 Identification of Soft Parameters

In the study conducted so far, all the soft parameters were either specified or mixed with hard parameters; these hard parameters were then treated as soft. For small thermal network the inclusion of the hard parameters as soft may be practical and thus considered realistic, but limited core storage locations will, in general, prevent a scheme of this type. As a result some means of identifying the soft parameters should be provided in addition to the manual means employed by the engineer directly. Use of sensitivity coefficients appears to be an attractive was for providing this information. However, a complete study will be required before its feasibility and practicality can be established.

5.1.4 Measured Temperature Data

The experimental data is to be provided by NASA/MSC if available. However, it is of particular importance to indicate that the accuracy of this information is essential for any network correction scheme. The study conducted heretofore employed calculated temperature and temperature derivative values which in essence were near-perfect. It remains to be seen what effects inaccurate temperatures will have on the accuracy of the parameter estimation. It is also clear that experimental temperature data must also be processed to eliminate obvious erroneous measurements and to smooth, perhaps in a least squares sense, the remaining data. It is assumed that NASA/MSC will provide the necessary processing and editing of the experimental interface data.

5.2 SYSTEMS PROGRAMMING CONSIDERATIONS

Meaningful correlation methods can become a reality only if a number of system programming considerations can be resolved. Some of these are discussed below.

5.2.1 Local Region Isolation

Local region isolation is a particular important systems problem that must be fully implemented before a large network correction technique can actually become a reality. It is hoped that by comparing the temperatures measured and theoretical, a programming scheme can be developed to do this.

5.2.2 Identification and Bounding of Soft Parameters

Theoretical aspects of soft parameter identification were discussed in a paragraph above. From a programming standpoint, a single subroutine to service any parameter correction scheme (within reasonable constraints) is being considered; this would thus allow standardization of identification and bounding of information. Input data would consist of any array of triplet values for each soft parameter. The first would identify the parameter while the other two would be the absolute lower and upper bounds. The subroutine would reorder the parameters into a desired sequence and print a dictionary identifying the soft parameter and its relative parameter number.

5.2.3 Input of Test Data

The subroutine to perform this function would be customized to the particular parameter correction subroutine which it services and also to the input format of the test data. It would require sufficient information so that it could correlate the test data with the analytical node points. A smoothing operations should be available if needed in order to insure smooth continuous functions and also to calculate temperature derivatives.

5.2.4 Parameter Correction Subroutine

No matter what method is used this subroutine will be required to initialize itself by obtaining matrix storage areas from dynamic core storage. The size requirements are immediately determinable from paragraph 5.2.2 and 5.2.3 above. In all probability, the parameter correction subroutine will internally call upon the identity and test data subroutines. It will then call upon lower level subroutines to form the matrices required by its particular method and call upon the SINDA matrix manipulation subroutines to perform any additional operations required.

5.2.5 Rerun and Verification

Since the test data is normally at a different frequency than the analytical computation step and usually several time slices are necessary before parameter correction is possible, automatic recalculation

of the network with the corrected parameters cannot be controlled by the subroutine discussed in paragraph 5.2.4 above. This function must be performed by an additional subroutine placed in the execution block which would correct the soft parameters, set time and temperatures back some previous value and logically route the sequence of operations to again call upon the network solution subroutine.

5.3 SUMMARY

These problem areas were presented here to indicate that a number of problems directly related to a correlation method and a number of systems programming considerations must be resolved before a practical correlation method for a large thermal network becomes a reality.

6.0 RECOMENDATIONS

It was the original intent of this Phase I study to make a specific recommendation of a correction method that indicated at least potentially the capability to correct parameters of a large thermal network, given temperature measurements. On the other hand, if the results were negative, then the conclusions must so indicate.

The results in many respects are quite promising and in other respects as indicated by the discussion on problem areas, less so. The chief concern at the present time is the accommodation of the sparsity of temperature measurements. Sufficient information has not been generated nor results obtained to make a realistic assessment on the magnitude of sparsity that can be handled. It is clear however that if a high percentage of temperature measurements are available or that the measurements are properly located, either the one-pass method or the sequential method can satisfactorily correct the parameters of a thermal network. The size of the thermal network remains an unanswered question. Other considerations such as core storage requirements, programming sophistication, numerical errors, etc. will govern which of the two methods should ultimately be employed. As a result, the following recommendation is made based upon the results of the present study and anticipated results.

It is recommended that both the one-pass method and the sequential method be studied further in Phase II until a more definite conclusion can be made. Simultaneously, other system aspects of the correlation methods will be developed in such a way that both methods can be handled directly.

7.0 REFERENCES

1. Graybill, F. A., An Introduction to Linear Statistical Models, Chapter 6, Vol. 1, McGraw-Hill Book Co., N. Y., 1961.

2. William, E. J., Regression Analysis, John Wiley & Son., N. Y., 1959.

3. Efroymson, M. A., "Multiple Regression Analysis", Mathematical Methods for Digital Computers, Edited by Ralston and Wilf, John Wiley & Son, 1960.

4. Hartley, H. O., "The Modified Gauss-Newton Method for the Fitting of Nonlinear Regression Functions by Least Squares", Technometrices, Vol. 3, No. 2, pp 269-280, May 1961.

5. Breaux, H. J., "On Stepwise Multiple Linear Regression", Ballistic Research Laboratories Report No. 1369, August 1967.

6. Giese, C. and McGhee, R. B., "Estimation of Nonlinear System States and Parameters by Regression Methods", 1965 Joint Automatic Control Conference, Rensselaer Polytechnic Institute, Troy, N. Y.

7. Rauch, H. E., "Least Squares Estimation with a Large Number of Parameters", 1967 Joint Automatic Control Conference, University of Pennsylvania.

8. Golub, G., "Numerical Methods for Solving Linear Least Squares Problems", Numerische Mathematik, 7, 206-216, 1965.

9. Marquardt, D. W., "An Algorithm for Least Square Estimation of Nonlinear Parameters", J. Soc. Indust. Appl. Math., 431-441, Vol. 11, No. 2, June 1963.

10. Smith, G. A., "The Theory and Application of Least Squares", NASA TM X-63127, December 1967.

11. Levenberg, K., "A Method for the Solution of Certain Non-Linear Problems in Least Squares", Quart. Appl. Math, 2, pp 164-168, 1964.

12. Toussaint, M., "Verification of the Thermal Mathematical Model for Artificial Satellites: A New Test Philosophy", AIAA Paper No. 67-304, presented at the AIAA Thermophysics Specialist Conference, 1967.

13. Allison, J. S., "The Parameter Estimation Problem in Mathematical Modelling", Report SAD 41, Department of Supply, Australian Defense Scientific Service, Weapons Research Establishment, November 1966.

14. Allison, J. S., "The Parameter Estimation Problem in Model Checking", Proceedings of the Third Australian Computer Conference, Canberra, May 16-20, 1966.

15. Ho, Y. C., "The Method of Least Squares and Optimal Filtering Theory", RM-3329-PR, October 1962, Rand Corporation.

16. Alford, M. W., Jr., "A Computationally Stable Noise-in-the-State Filtering Algorithm", AIAA Paper No. 68-887, presented at the AIAA Guidance, Control and Flight Dynamics Conference, August 1968.

17. Aoki, Masanao, Optimization of Stochastic Systems, Topics in Discrete-Time Domain, Chapter 5, Academic Press, New York 1967.

18. Nickle, C. E., "Linear Error Analysis and Filtering by Recursive Least Squares", Technical Memorandum HSM-M58-67, June 1967. Space Division, Chrysler Corporation, Hunstville Operation.

19. Mowery, V. O., "Least Squares Recursive Differential-Correction Estimation in Nonlinear Problems", IEEE transactions on Automatic Control, Vol. AC-10, No. 4, pp 399-407, October, 1965.

20. Ho. H. C., "A Bayesian Approach to Problems in Stochastic Estimation and Control", IEEE transactions on Automatic Control, pp 333-339, October 1964.

21. Bellman, R. E., Kagiwada, H. H., Kalaba, R. E., and Sridhar, R., "Invariant Imbedding and Nonlinear Filtering Theory", The Journal of Astonautical Sciences, Vol. 13, No. 3, pp 110-115, June 1966.

22. Detchmendy, D. M., and Sridhar, "Sequential Estimation of States and Parameters in Noisy Nonlinear Dynamic Systems", Journal of Basic Engineering, Transactions of the ASME, pp 362-368, June 1966.

23. Sridhar, R., Bellman, R. E., and Kalaba, R. E., "Sensitivity Analysis and Invariant Imbedding", Memorandum RM-4039-PR, The Rand Corporation, March 1964.

24. Bellman, R., Kagiwada, H. H., and Kalaba, R. E., "Quasilinearization, Systems Identification, and Prediction", Memorandum RM-3812-PR, The Rand Corporation, August 1963.

25. Prasanna-Kumar, K. S., and Sridhar, R., "On the Identification of Control Systems by the Quasi-Linearization Method", IEEE transactions on Automatic Control, pp 151-154, Vol. AC-9, April 1964.

26. Kalman, R. E., "A New Approach to Linear Filtering and Prediction Problems", Journal of Basic Engineering, Transaction of ASME, pp 35-45, March 1960.

27. Kalman, R. E., and Bucy, R. S., "New Results in Linear Filtering and Prediction Theory", Journal of Basic Engineering, Transaction of ASME, pp 95-108, March 1961.

28. Kalman, R. E., "New Methods and Results in Linear Prediction and Filtering Theory," RIAS TR 61-1, Presented at the Symposium on Engineering Application of Random Function Theory and Probability, Purdue University, November 1960.

29. Lear, W. M., "Derivation, Discussion and Application of Kalman Filtering Techniques", TRW Report No. 7221.11-18, August 1967.

30. Browne, B. H., Jr., "Statistical Estimation and Elimination of Thermal Network Model Errors", ASME Paper 68-HT-48, 1968.

31. Tomovic, R., "Sensitivity Analysis of Dynamic Systems", McGraw-Hill Book Company, Inc., 1963.

32. Meissinger, H. F., "The Use of Parameter Influence Coefficients in Computer Analysis of Dynamic Systems", Simulation, August 1964, Vol. 4, No. 10, pp 53-63.

33. Wilkinson, J. H., "The Solution of Ill-Conditioned Linear Equations", Mathematical Methods for Digital Computers, Vol. 2, Edited by A. Ralston and H. S. Wilf, John Wiley & Sons, 1967.

34. Smith, L. H., and Powers, C. S., "Oracle-MAFIA Program", TRW Report 05952-6178-T000, 27 September 1967.

35. Lewis, D. R., Gaski, J. D., and Thompson, L. R., "Chrysler Improved Numerical Differencing Analyzer for 3rd Generation Computers, CINDA 3G", TN-AP-67-287, October 20, 1967.

APPENDIX A FIVE- AND TWENTY-NODE THEORETICAL MODELS

The parameter estimation (correlation techniques) methods were studied in detail by the use of the 5- and 20-node models illustrated in Figures A-1 and A-2. These particular models were chosen because the mechanism of heat conduction and thermal radiation are well-represented.

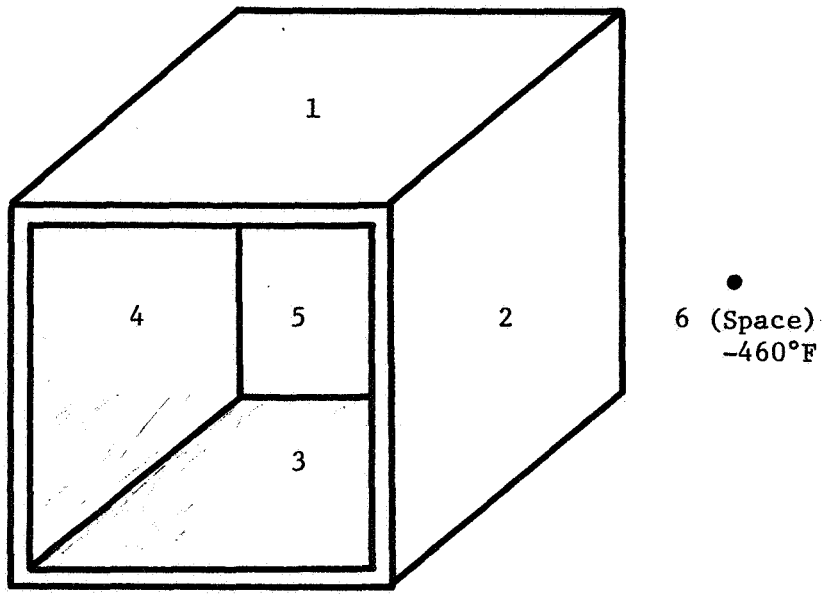
The five-node model provides a preliminary test of the several correlation methods that have passed the initial and secondary screening. The twenty-node model provides a more severe test of the techniques studied with the five-node model and provides some insight on the problems of model size; for example numerical errors may be of particular importance.

A.1 FIVE-NODE MODEL

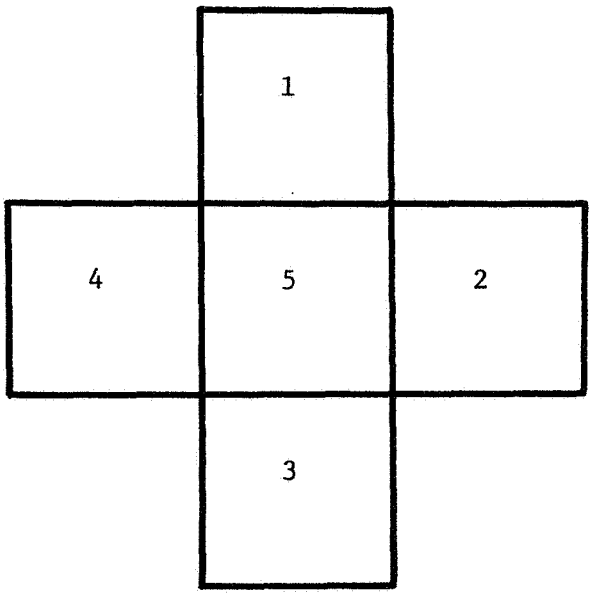
The five-node model as pictured in Figure A-1 can be described as a hollow cube with one face open to space. This model allows a good mixture of conduction and radiation exchange and provides a reasonable test of the correlation methods since the radiation exchange is an order of magnitude smaller than the heat conduction flow from one surface to another. The thermal properties and environmental conditions are tabulated in Table A-2 and the transient response of each nodal temperature starting from the steady state temperature is tabulated in Table A-3.

A.2 TWENTY-NODE MODEL

The twenty-node model as pictured in Figure A-2 is a larger version (more nodes) of the hollow-cube of Figure A-1. This model provides the same characteristics of the smaller five-node model with the additional complexities associated with a larger system that magnifies the computer storage, numerical errors, and programming subtleties. The thermal properties and environmental conditions are tabulated in Table A-4; the nominal parameter values are tabulated in Table A-5 and the transient response of a few selected nodal temperature starting from the steady state temperature is tabluated in Table A-6.

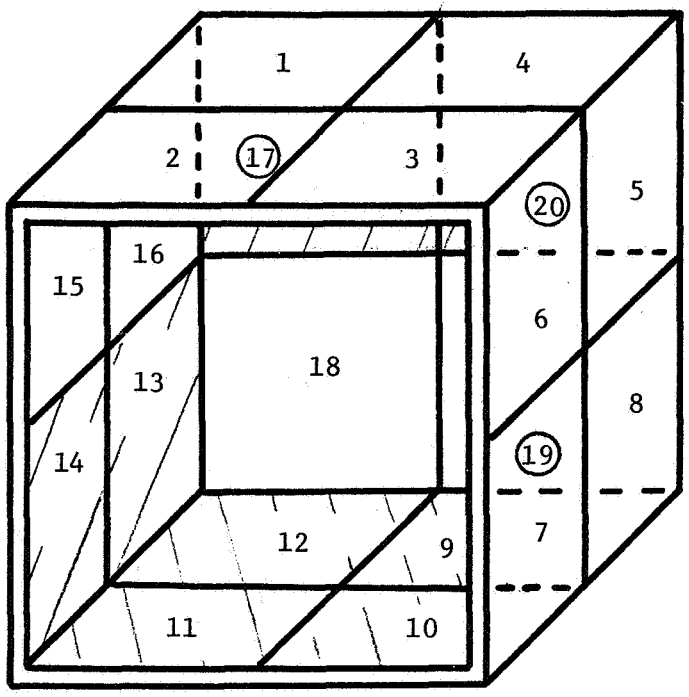


(a) Five-Node Model

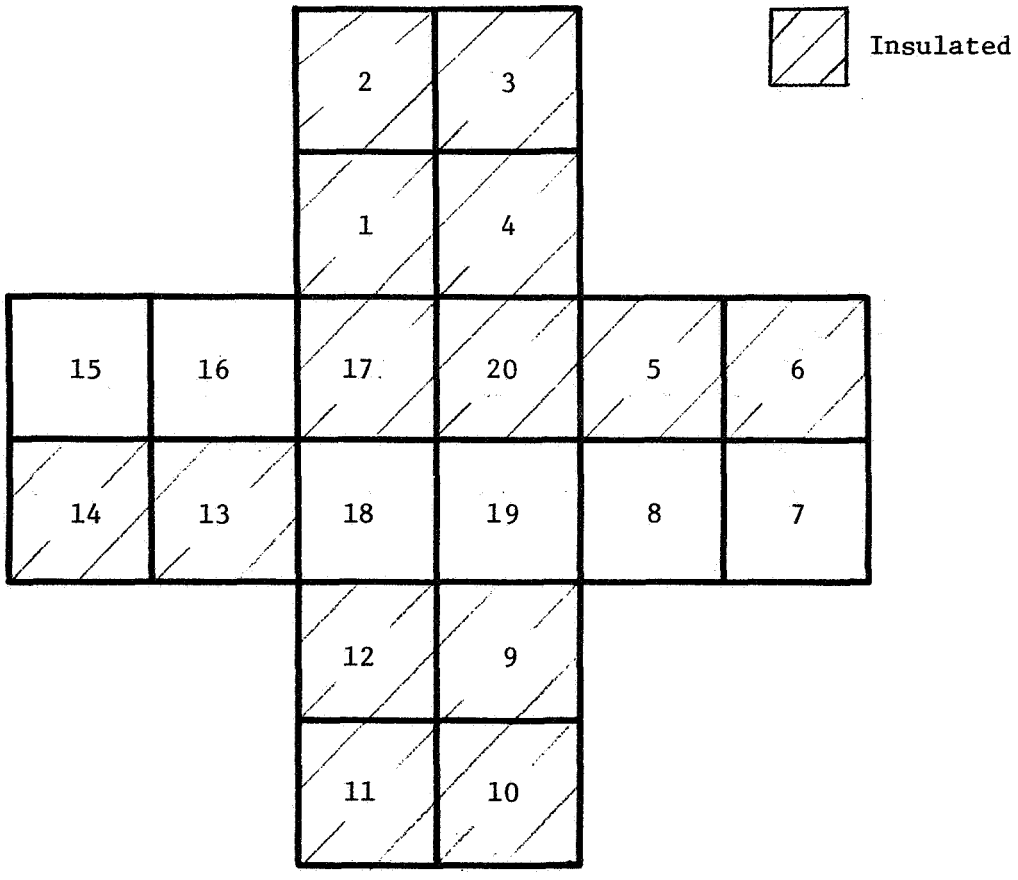


(b) Five-Node Model Folded Inside Out

FIVE A-1 FIVE-NODE HOLLOW CUBE MODEL



(a) Twenty-Node Model



(b) Twenty-Node Model Folded Inside Out

FIGURE A-2 TWENTY-NODE HOLLOW CUBE MODEL

TABLE A-1

FIVE-NODE MODEL, THERMAL-GEOMETRICAL PROPERTIES AND EXTERNAL INPUT

Characteristics \ Surface	1	2	3	4	5	All
Emissivity (Dimensionless						
External	1	0	1	0	1	-
Internal	1	1	1	1	1	-
Heat Input (BTU/hr)						
Steady State	100	150	0	0	300	-
Transient	100	0	0	0	0	-
Initial Temp. for Trans. (°F) (From Steady State Values)	64.9	114.6	36.4	62.1	102.6	-
Density (γ, lb/ft^3)						174
Heat Capacity, C_p (BTU/lb °R)						.23
Thermal Conductivity, k, (BTU/hr °F/ft)						100
Dimensions (ft)						1'x1'x.005'
Shape Factors Between Surfaces (F_{ij})						0.2
Sink Temperature (°F)	-460					

TABLE A-2

FIVE-NODE MODEL, NOMINAL PARAMETER VALUES

Coefficients	Value	Coefficients	Value	Coefficients	Value
a_{12}	0.5 BTU/hr°F	b_{15}	0.2 (no dimen.)	b_{56}	1.2 (no dimen.)
a_{14}	0.5 BTU/hr°F	b_{23}	0.2 (no dimen.)	C_1	0.2 BTU/hr
a_{15}	0.5 BTU/hr°F	b_{24}	0.2 (no dimen.)	C_2	0.2 BTU/hr
a_{23}	0.5 BTU/hr°F	b_{25}	0.2 (no dimen.)	C_3	0.2 BTU/hr
a_{25}	0.5 BTU/hr°F	b_{26}	0.2 (no dimen.)	C_4	0.2 BTU/hr
a_{34}	0.5 BTU/hr°F	b_{34}	0.2 (no dimen.)	C_5	0.2 BTU/hr
a_{35}	0.5 BTU/hr°F	b_{35}	0.2 (no dimen.)	Q_1	100 BTU/hr
a_{45}	0.5 BTU/hr°F	b_{45}	0.2 (no dimen.)	Q_2	150 BTU/hr
b_{12}	0.2 (no dimen.)	b_{46}	0.2 (no dimen.)	Q_3	0 BTU/hr
b_{13}	0.2 (no dimen.)	b_{16}	1.2 (no dimen.)	Q_4	0 BTU/hr
b_{14}	0.2 (no dimen.)	b_{36}	1.2 (no dimen.)	Q_5	300 BTU/hr

TABLE A-3

FIVE-NODE MODEL, TRANSIENT RESPONSE*

Time (hr)	1	2	3	4	5
0	64.9000	114.600	36.4000	62.1000	102.6000
0.01	64.5399	107.332	36.0388	61.8077	88.9352
0.02	63.5530	100.527	35.0404	60.9943	77.6165
0.03	62.1140	94.1494	33.5730	59.7715	68.0930
0.04	60.3527	88.1628	31.7631	58.2266	59.9625
0.05	58.3656	82.5323	29.7060	56.4291	52.9261
0.06	56.2249	77.2253	27.4744	54.4349	46.7583
0.07	53.9843	72.2120	25.1234	52.2891	41.2869
0.08	51.6841	67.4659	22.6951	50.0286	36.3793
0.09	49.3543	62.9629	20.2213	47.6834	31.9323
0.10	47.0173	58.6818	17.7266	45.2782	27.8648
0.11	44.6896	54.6038	15.2293	42.8330	24.1130
0.12	42.3831	50.7118	12.7434	40.3642	20.6263
0.13	40.1067	46.9909	10.2794	37.8853	17.3640
0.14	37.8665	43.4277	7.84513	35.4074	14.2936
0.15	35.6669	40.0101	5.44614	32.9392	11.3887
0.16	33.5107	36.7275	3.08657	30.4880	8.62788
0.17	31.3997	33.5702	.769264	28.0596	5.99363
0.18	29.3348	30.5295	-1.50392	25.6588	3.47152
0.19	27.3162	27.5976	-3.73185	23.2890	1.04961
0.20	25.3440	24.7676	-5.91397	20.9533	-1.28201
0.21	23.4176	22.0329	-8.05015	18.6537	-3.53172
0.22	21.5363	19.3879	-10.1406	16.3920	-5.70654
0.23	19.6992	16.8271	-12.1858	14.1693	-7.81245
0.24	17.9052	14.3460	-14.1864	11.9865	-9.85455
0.25	16.1534	11.9399	-16.1432	9.84391	-11.8372
0.26	14.4425	9.60494	-18.0570	7.74188	-13.7644
0.27	12.7715	7.33739	-19.9289	6.68038	-15.6393
0.28	11.1391	5.13385	-21.7598	3.65926	-17.4649
0.29	9.54414	2.99120	-23.5507	1.67820	-19.2439
0.30	7.98557	.906536	-25.3027	-.263203	-20.9785

* Nominal parameter values as indicated in Table A-2

TABLE A-4

TWENTY-NODE MODEL, THERMAL-GEOMETRICAL PROPERTIES AND EXTERNAL INPUT

Surface / Characteristics	1,2,3,4,	5,6	7,8	9,10,11,12	13,14	15,16	17,20	18,19
Emissivity (Dimensionless)								
External	1	0	0	1	0	0	1	1
Internal	1	0	1	0	0	1	0	1
Heat Input (BTU/hr)								
Steady State	100	150	150	0	0	0	300	300
Transient	100	0	0	0	0	0	0	0
Dimensions (ft)	All Surfaces 1/2' x 1/2' x .005'							
Thermal Properties	Same as five-node model							
Density								
Heat Capacity								
Conductivity								

Shape Factors

$F_{i,21} = 1$, i = 1,2,3,4,9,10,11,12,17,20

$F_{7,21}$; $F_{15,21} = .295$: $F_{18,21}$; $F_{19,21} = 1.2$: $F_{8,21}$; $F_{16,21} = .105$

$F_{7,15}$; $F_{8,16} = .048$: $F_{7,16}$; $F_{8,15} = .036$; $F_{7,18} = 0.03$: $F_{7,19}$; $F_{8,18} = .035$

$F_{8,19} = 0.2$; $F_{15,18}$; $F_{15,19}$; $F_{16,19} = .02$; $F_{16,18} = .04$

TABLE A-5

TWENTY-NODE MODEL, NOMINAL PARAMETER VALUES

a_{12}	0.5 BTU/hr°F	$a_{11,12}$	0.5 BTU/hr°F	$b_{17,21}$	1	C_1	.05 BTU/hr
a_{14}	0.5 BTU/hr°F	$a_{11,14}$	0.5 BTU/hr°F	$b_{20,21}$	1	C_2	.05 BTU/hr
a_{16}	0.5 BTU/hr°F	$a_{12,13}$	0.5 BTU/hr°F	$b_{7,21}$	.295	C_3	.05 BTU/hr
a_{17}	0.5 BTU/hr°F	$a_{12,18}$	0.5 BTU/hr°F	$b_{15,21}$	.295	.	.05 BTU/hr
a_{23}	0.5 BTU/hr°F	$a_{13,14}$	0.5 BTU/hr°F	$b_{18,21}$	1.2	.	.05 BTU/hr
$a_{2,15}$	0.5 BTU/hr°F	$a_{13,16}$	0.5 BTU/hr°F	$b_{19,21}$	1.2	.	.05 BTU/hr
a_{34}	0.5 BTU/hr°F	$a_{13,18}$	0.5 BTU/hr°F	$b_{8,21}$	.105	.	.05 BTU/hr
a_{36}	0.5 BTU/hr°F	$a_{14,15}$	0.5 BTU/hr°F	$b_{16,21}$	.105	C_{18}	.05 BTU/hr
a_{45}	0.5 BTU/hr°F	$a_{15,16}$	0.5 BTU/hr°F	$b_{7,15}$	.048	C_{19}	.05 BTU/hr
$a_{4,20}$	0.5 BTU/hr°F	$a_{16,17}$	0.5 BTU/hr°F	$b_{8,16}$	.048	C_{20}	.05 BTU/hr
a_{56}	0.5 BTU/hr°F	$a_{17,18}$	0.5 BTU/hr°F	$b_{7,16}$	.036	Q_1	100 BTU/hr
a_{58}	0.5 BTU/hr°F	$a_{17,20}$	0.5 BTU/hr°F	$b_{8,15}$	.036	Q_2	100 BTU/hr
$a_{5,20}$	0.5 BTU/hr°F	$a_{18,19}$	0.5 BTU/hr°F	$b_{7,18}$	.03	Q_3	100 BTU/hr
$a_{6,7}$	0.5 BTU/hr°F	$a_{19,20}$	0.5 BTU/hr°F	$b_{7,19}$	.035	Q_4	100 BTU/hr
a_{78}	0.5 BTU/hr°F	$b_{1,21}$	1 (no dimen.)	$b_{8,18}$	.035	Q_5	150 BTU/hr
$a_{7,10}$	0.5 BTU/hr°F	$b_{2,21}$	1 (no dimen.)	$b_{8,19}$	0.2	Q_6	150 BTU/hr
a_{89}	0.5 BTU/hr°F	$b_{3,21}$	1 (no dimen.)	$b_{15,18}$	0.2	Q_7	150 BTU/hr
$a_{8,19}$	0.5 BTU/hr°F	$b_{4,21}$	1 (no dimen.)	$b_{15,19}$	0.2	Q_8	150 BTU/hr
$a_{9,10}$	0.5 BTU/hr°F	$b_{9,21}$	1 (no dimen.)	$b_{16,19}$	0.2	Q_{17}	300 BTU/hr
$a_{9,12}$	0.5 BTU/hr°F	$b_{10,21}$	1 (no dimen.)	$b_{16,18}$	0.04	Q_{18}	300 BTU/hr
$a_{9,19}$	0.5 BTU/hr°F	$b_{11,21}$	1 (no dimen.)			Q_{19}	300 BTU/hr
$a_{10,11}$	0.5 BTU/hr°F	$b_{12,21}$	1 (no dimen.)			Q_{20}	300 BTU/hr

APPENDIX B. ERROR ASSESSMENT IN THERMAL SYSTEMS

B.1 INTRODUCTION

Any of the methods that is utilized to correct the parameters of a thermal network must compare predicted and measured temperatures. If these temperatures were known to be precise, it would be a simple matter to identify those which do not compare exactly or those which fall outside a specified tolerance band.

Unfortunately, temperature predictions are not precise but are subject to variances in the input data and inaccuracies of the computational method. In a similar vein temperature measurements are not exact but are subject to instrumentation, data processing, and recording error. Thus any comparison between prediction and measurement must account for the effects of these uncertainties in order to ascertain out-of-tolerance temperatures.

Successful assessment of the uncertainty and approximation effects on temperature will require an error examination in several categories: (1) uncertainties and approximations in thermal analysis; (2) uncertainties associated with the ground-based environment; (3) uncertainties in data acquisition; and (4) uncertainties associated with the defined space environmental conditions. Category (4) is required if predicted and flight information are to be correlated. Categories (2) and (3) become necessary when predicted and environmental test information are to be compared.

In the presentation to follow, the four categories are discussed to some extent but the emphasis has been placed upon the uncertainties and approximations in thermal analysis. The approximations include an investigation on several numerical integration schemes that are available on CINDA-3G.

Some of the information contained in the sections to follow is also reported in other References (B-1, B-2). Some of the material has been expanded to include more recent information.

B.2 UNCERTAINTIES AND APPROXIMATIONS IN THERMAL ANALYSIS

The mathematical model and the solution technique may be precise but the input information to these mathematical models are not known exactly. In general, a nominal value is utilized but an uncertainty bound should also be included. The term uncertainty as used here is the one employed by Kline and McClintock[B-3*]. These authors define uncertainty as a possible value the error might have.

Apart from the uncertainties of the input information are the approximations that must be utilized to translate the physical system to a mathematical model and to obtain useful results from the governing equations which are solved numerically. The errors due to these approximations are subject to many considerations but perhaps the major ones are the cost, the time, and the engineer who generates the mathematical model. In the present thermal network correction study, the approximations are considered to be small.

B.2.1 Uncertainty Interval in the Thermal-Geometrical Characteristics

Evaluation of the uncertainty interval for a variable is at best a difficult task. The uncertainty of the variable must include the best estimate of the true value as well as the magnitude of the error. The use of standard deviation to describe uncertainties appears to be unrealistic because single-sample experiments are often used in lieu of multiple-sample experiments. Only a limited amount of information on the uncertainty of variables used for spacecraft thermal analysis appears to be available. The present effort being conducted at NASA/MSC will provide considerably more information[B-4]. Until more thermal property uncertainty information becomes available, it will be necessary to estimate the magnitude of the uncertainties wherever the void exists.

The input information to a mathematical model may be separated conveniently into three categories: (1) thermophysical characteristics of the test article; (2) thermal environment of a ground-based facility; and (3) external environment in space.

---

* Superscript numbers preceded by the letter B refer to the references at the end of the Appendix.

B.2.1.1 Thermophysical Properties

The thermophysical properties include those under the general heading of thermal radiation properties such as emittance and absorptance and others such as thermal conductivity and density under the heading, heat transfer and physical properties.

B.2.1.1.1 Thermal Radiation Properties

The thermal radiation properties of materials which include coatings are subject to a number of errors that contribute to the uncertainties of the property values. An obvious source of uncertainty is in the measurement process. The experimental errors depend upon the geometry of the system, the physical characteristics of the detectors and optical elements, the thermal environment of the sample, etc.[B-5] It is stated in Reference B-5 that a general value of the uncertainty interval of any sample cannot be given but for purposes of an uncertainty study here the solar absorptance α_s, and the hemispherical emittance are estimated to have an uncertainty interval of ± 0.02 subject of course to the constraint that $0 \leq \frac{\alpha_s}{\varepsilon_h} \leq 1.0$. The uncertainty of the thermal radiation property of materials found on a spacecraft is increased by other factors such as:

(1) Poor quality control (which allows wide variance in the surface finish);

(2) Sample differences;

(3) Normal manufacturing and handling processes;

(4) Corrosive atmosphere before launch (or during storage); and

(5) Exposure to a deleterious environment during test such as a carbon-arc source with its ultra-violet band, during ascent and during the period in space.

The uncertainty interval due to the factors listed above is difficult to estimate, but it is probably greater than ± 0.01. In addition, it should be recognized that the uncertainty of the spectral intensity distribution of the incident energy in general increases the coating property variances of the irradiated surfaces. Needless to say, if these coatings on the external surfaces have a flat response, the spectral intensity distribution of the incoming external energy source is not important.

The thermal radiation properties are also temperature sensitive.[B-6] If these values are known and utilized in the analysis, then errors due to temperature changes do not occur. Unfortunately, a wealth of information in this area is not available, particularly at low temperatures. Below -50°F, the measurement errors can be serious.[B-7] The results of Reference B-8 indicate that below -50°F, the change in emittance with temperature may be large. However, it is not unusual to find surfaces that reach temperatures much below -50°F; for example, a solar array may reach a level as low as -250°F and higher than 200°F.

Based upon the considerations discussed above, the uncertainty interval for the hemispherical emittance and the solar absorptance is estimated to be $\pm$.03.

B.2.1.1.2 Interface Conductance

The interface conductance in structural joints and/or at component mounting interfaces is subject to many factors which from practical considerations is beyond precise control. Joint conductance variations of supposedly identical joints are considerable. A number of analytical studies and experimental programs are reported in technical literature. Reference B-9 lists approximately 200 references on interface conductance and related areas. Most of these reported studies are based on ideal interfaces which are different from joints used in spacecraft design.[B-10 to B-14]

A considerable amount of experimental results of joints used in aircraft is available as typified by References B-15, B-16, and B-17. These tests were conducted in air. Some experimental results in a vacuum of practical joints have been reported.[B-18 to B-21] The variance of a standard joint conductance can only be realistically estimated for a specific joint based on experimental data. Contact conductance of joints in the Apollo spacecraft is reported in References B-20 and B-21. It is not unusual to find that the joint conductance of "identical" joints differ more than 100%; perhaps for uncertainty studies, a variance in the range $\pm$25% to $\pm$50% is realistic.

B.2.1.1.3 Thermal and Physical Properties

The thermal and physical property values (thermal conductivity, heat capacity, and density) used in thermal analysis generally are

handbook values. Quite often these properties are not known for the specific material to be used; the use of "best" available information leads to significant errors. Even "batch" to "batch" differences may be appreciable. Aside from the errors due to material differences, measurement errors are also significant.

The use of the accurate Bunsen-Type calorimeter[B-22] to measure the heat capacity of a material yields values that have a variance of ±3%.[B-23] The accuracy of thermal conductivity values depends upon many factors such as the type of material to be tested. The guarded hot plate method[B-24] has a reported accuracy of ±5% for low temperature thermal conductivity tests in the range of -320°F to +50°F.

If the property values as typically found in literature are used, the variance is estimated to be ±10%.

B.2.1.1.4 Insulation

Insulation as defined here refers to multilayer insulation which is found in several types such as crinkled Mylar and dimpled Mylar (Dimplar).[B-25] Although the use of insulation on spacecraft is commonplace, design problems arise because of the lack of precise information on the insulation characteristics. Effective conductance through the insulation is not known precisely because important factors such as packing density and edge effects cannot be accurately controlled. As a result, the prediction of the effective conductance with a small variance is beyond the present state-of-the-art. At present, the thermal control system in essence utilizes the insulation system to minimize the heat flow by specifying a maximum allowable effective conductance; specification of both an upper and a lower bound on the insulation characteristics would lead to prediction difficulties. The variance of the insulation system is subject to a large number of factors, many of which are beyond precise control. In general, the variance can be larger than ±50% although it is reported in Reference B-26 for a Dimplar insulation system that an error in prediction was less than ±15% and repeatability was within ±5% of the average.

Trapped gases within the insulation could lead to prediction difficulties, especially in an environmental facility with only a low vacuum capability. It should be recognized that the pressure within

the insulation will be higher than the pressure within the chamber. Unfortunately, a technique to measure insulation pressure in the free-molecular region is not available at the present time.[B-27]

B.2.1.2 Geometric Characteristics

Dimensions of the test article are known quite accurately (better than .01%); as a result dimensional uncertainties of the total test article have very little effect on the uncertainties of the temperature. However, the use of the lumped parameter technique will increase the dimensional errors, particularly if the physical system is composed of complex geometric configuration. This will be discussed further in a later paragraph concerning thermal resistance. In a similar vein; the shape factor from one discrete area to another is subject to dimensional inaccuracies but other factors may be more important; this will also be discussed subsequently.

B.2.1.3 Characteristics of a Louver System

An important component of an active thermal control system that controls part of the heat flow from the spacecraft is a louver.[B-28] Prediction of the louver thermal radiating characteristics is difficult; however, analytical treatments are available in literature.[B-29 to B-33] In general, experimental data is more reliable. The problem of prediction is magnified considerable if an external source is incident on the louver system. Some experimental data without incident energy are available.[B-30, B-33, & B-34] The result of Reference B-34 indicate that the variance of the experimental information is approximately $\pm$10%.

B.2.2 Approximation in the Analytical Method

A spacecraft is not a simple geometric object with surface characteristics that may be described by accurate analytical expressions; on the contrary, a spacecraft is a complex system composed of a structural shell of various shapes and design enveloping, to protect, components "packed" within the confines of the shell. Each component, it may be added, represents a thermal design challenge which perturbs the placement of these components within the spacecraft (subject, of course, to all spacecraft constraints).

An attempt to describe the thermal interchange within an enclosure with the multitude of components and support structure for a distributive

system would be almost prohibitive; the difficulties encountered would be almost prohibitive; the difficulties encountered in the solution of the thermal radiation-conduction coupled expressions would be beyond present numerical techniques if accuracy of a high order were desired. The alternative and one that is commonly employed today is the lumped parameter system.[B-33] Thus, analytical sophistication to improve thermal prediction of a spacecraft are constrained by the discrete area approximation. Less complex systems are not subject to this constraint.

B.2.2.1 Lumped Parameter System

To a large extent, translation of the physical system to the topological model consisting of a network with resistors and capacitors[B-35,B-36] requires engineering judgement. Quite often budget limitations coupled with core storage capacity of the computer dictate the number of nodes that may be utilized. As a result the discrete areas larger than desired are often used and transient solutions with less than maximum accuracy are found to minimize computer run-time. This criticism should be tempered by the fact that high accuracy is often secondary for many spacecraft thermal control design. When a close temperature tolerance system is required, it becomes necessary to examine those errors that are ordinarily assumed to be small.

B.2.2.1.1 Truncation Error

Truncation error is defined here to mean the temperature difference between the exact solution of the distributive system and the exact solution of the finite-difference expression which may be one of several types such as explicit, implicit, and laternating direction.[B-37] For the explicit forward difference technique, the truncation error E may be expressed as:[B-37]

$$E \simeq \frac{\Delta t}{2}\frac{\partial^2 T}{\partial t^2} - \frac{(\Delta x)^2}{12}\frac{\partial^4 T}{\partial x^4} - \frac{(\Delta y)^2}{12}\frac{\partial^4 T}{\partial y^4} + \mathcal{O}(\Delta t)^2 + \mathcal{O}(\Delta x)^4 + \mathcal{O}(\Delta y)^4 \qquad \text{(B-1)}$$

Equation (B-1) indicates that the error due to the finite difference approximation is a function of the grid size, the time increment, and the temperature distribution which is the unknown that is to be found. No attempt will be made to estimate this error at this time for 'typical'

spacecraft heat transfer problems. However, indications of the truncation error are given in Reference B-38 which examines a semi-infinite slab numerically solved by an implicit method discussed in Reference B-39 and in Reference B-40 (or Reference B-41) which employs a semi-discrete analog (discrete spatially and continuous in time) to examine a finite slab. Magnitudes of discretization errors in a finite difference solution of the heat flow equation in a symmetric slab are presented in Reference B-42.

In addition to the error at the nodal points, an error due to interpolation between nodes arises. Usually, the temperature variation between nodes is assumed to be linear; the maximum absolute value of this error due to linear interpolation, E_I, may be expressed as:[B-35]

$$|E_I| \max \leq \frac{\Delta x^2}{8} \left| \frac{\partial^2 T}{\partial x^2} \right| \qquad \text{(B-2)}$$

B.2.2.1.2 Non Uniform Local Heat Flux

Another approximation error which is due to discretizing is the assumption of constant radiosity for the discrete areas. The inaccuracies can be expected to effect the temperature level and the temperature distribution which is shown by equation (B-1) affects the truncation error. The influence of non-uniform local heat flux on overall heat transfer between a gray differential area parallel to a gray infinite plane was studied in Reference B-43. The assumption of uniform local heat flux appears to be adequate insofar as this geometry and the overall heat flux calculations are concerned but its effects on the temperature distribution remains to be studied. Thermal radiation exchange in three systems, parallel plates, adjoint plates, and circular disks is studied in Reference B-44. Significant variations in the local heat transfer along a surface are reported.

The above discussion merely serves to indicate in spacecraft thermal analysis that not only must the conductive aspects be considered in arriving at the grid size, but the effects of non-uniform heat flux must also be examined.

B.2.2.2 Methods of Radiation Analysis

A brief review of analytical methods related to thermal radiation analysis is presented in Reference B-19. The number of technical papers and books in this area is considerable; no attempt is made here to indicate these references. Perhaps a single best source is Reference B-45 and a very interesting one on radiation interchange factors is Reference B-46. The difficulties encountered in assessing improved analytical techniques for spacecraft thermal analysis are discussed in Reference B-1. Errors in the input information and the inaccuracies of the environmental facilities including the data acquisition system subordinate the temperature correlation.

B.2.2.3 Stefan-Boltzmann Constant

The Stefan-Boltzmann constant employed in the expression for radiation exchange is generally used in heat transfer calculations without regard to its accuracy. Admittedly for most spacecraft thermal considerations, the uncertainty interval of the Stefan-Boltzmann constant is not sufficient to warrant undue examination. However, this uncertainty is important, say in the design of a radiometer. It is reported in technical literature that the variance of the Stefan-Boltzmann constant is about $\pm$0.6%.[B-47 to B-49]

B.2.3 Numerical Technique

The set of ordinary non-linear differential equations that arise from the lumped parameter system can be solved realistically only by a numerical method coupled with a high speed computer. Accuracy and computer time considerations have prompted a number of studies in the numerical solution of the parabolic equation and the set of ordinary non-linear differential equations associated with the lumped parameter system.[B-37, B-50 to B-53] There does not appear to be a single best method since many factors such as boundary conditions, accuracy, core storage and computer run time must all be considered. A comparison of several finite difference formulations is presented in Reference B-37. In Reference B-53, six numerical methods were applied to a 3-node system. The methods studied were: Euler, Heun, Runge-Kutta Fourth Order, Adams-Bashforth Prediction, Milner-Simpson Predictor/Corrector (constant step-size) and Heun (variable step-size). (The reader should refer to Reference B-53 for Details).

In the present study, no attempt was made to compare in a general sense the various numerical integration techniques for errors that arise from: (1) temperature truncation errors, arising from the discrete temperature changes that occur at each node in each time step, for which average values of temperature-dependent variables must be estimated; (2) time truncation errors, arising from the use of discrete time steps in the transient calculation, for which average values of time-dependent variables must be estimated; (3) convergence errors, arising from the use of an iterative method of solution for heat transfer for connected special nodes such as zero capacity nodes; and (4) arithmetic truncation errors, arising from accumulation of round-off errors and from the loss of significant figures that occur when numbers having large difference in values are involved in a calculation, or differences between number of similar magnitude are calculated. A discussion of these errors is presented in Reference B-54. Round-off errors are discussed in Reference B-55 and B-56. Rather, in the present study, integration routines presently available in CINDA-3G were studied.[B-57] These subroutines are denoted as follows:

(1) Steady State

CINDSS -- "Block" iterative method (a set of old temperatures replaced by new (just calculated) temperatures

CINDSL -- Successive point iteration (newest temperature available is always utilized)

(2) Transient

CNFRWD -- Explicit forward differencing (Euler technique)[B-58,B-59]

CNBACK -- Implicit backward differencing (standard implicit method)[B-37]

CNFWBK -- Implicit forward-backward (Crank-Nicholsen method)[B-57]

These CINDA-3G subroutines were evaluated by using the 5- and 20-node mathematical models described in Appendix A. Comparison of CINDSS and CINDSL is presented in Table B-1. From an accuracy standpoint, both methods yielded almost identical results. From a computer run-time standpoint, the models (5- and 20-node models) were two small to distinguish small differences in run-time.

TABLE B-1

COMPARISON OF CINDSS AND CINDSL

Five Node Model			Twenty-Node Model					
Node	CINDSS	CINDSL	Node	CINDSS	CINDSL	Node	CINDSS	CINDSL
	T(°F)	T(°F)		T(°F)	T(°F)		T(°F)	T(°F)
1	64.9220	64.9222	1	80.1167	80.1169	11	-24.0483	-24.0480
2	114.569	114.569	2	61.0493	61.0494	12	9.34294	9.34319
3	36.4349	36.4352	3	105.356	105.356	13	62.6028	62.6033
4	62.1131	62.1135	4	115.210	115.210	14	29.6337	29.6342
5	102.616	102.616	5	271.639	271.640	15	50.3469	50.3472
6	-460	-460	6	290.017	290.017	16	86.5191	86.5194
			7	193.056	193.056	17	149.426	149.426
			8	202.329	202.329	18	124.916	124.916
			9	40.2435	40.2437	19	151.270	151.270
			10	12.6997	12.6999	20	179.001	179.001
						21	-460	-460

The CNFRWD, CNBACK, and CNFWBK were studied on both the 5- and 20-node models. However, only the results for the five-node model are presented (Table B-2) since the results for the 5- and 20-node models are similar. Comparison of the three integration subroutines show that the CNFWBK temperatures lag the CNFRWD temperatures and that the CNFWBK temperatures oscillates for the time steps that were chosen. The time step (.5) for both the CNFWBK and CNBACK was an order of magnitude larger than the time step (.085) for CNFWRD. If the .5 time step were reduced, the results for CNFWBK and CNBACK are expected to be better. The oscillatory behavior of CNFWBK is believed to be caused by the radiation exchange. It should be noted that CNFWBK was originally formulated to handle linear conductors, not non-linear elements as present in the five- and twenty-node models. A more detailed study of these routines is necessary in order to resolve this problem.

B.3 UNCERTAINTIES ASSOCIATED WITH THE ENVIRONMENTAL FACILITY

The rapid expansion of the number of environmental facilities coupled with rapid changes in the state-of-the-art solar radiation simulation has created a variety of test facilities to fulfill a number of diverse requirements.[B-60, B-61] As a result, each test facility must be examined relative to its capabilities to provide the type as well as the accuracy of the thermal environment.

Without the specifics of a particular facility, the uncertainties of the thermal environment can only be discussed in generalities. It should also be recognized that the uncertainties of the environmental facilities are in many instances coupled with the data acquisition uncertainties which is discussed in a subsequent section. The discussion to follow will not be detailed since many published documents are available on this subject.[B-62 to B-72]

B.3.1 Test Philosophies

A discussion on test philosophies is interjected here to explain the usage of the term uncertainties as applied to environmental testing. When a test article is subjected to an environmental test, generally, one of the following two objectives is planned: (1) verification of the "hardware"; and (2) verification of the mathematical model. The

TABLE B-2

COMPARISON OF CNFRWD, CNFWBK, AND CNBACK, FIVE-NODE MODEL

Time (hr)	Node 1			Node 3			Node 5		
	CNFRWD	CNFWBK	CNBACK	CNFRWD	CNFWBK	CNBACK	CNFRWD	CNFWBK	CNBACK
0	64.9000	64.9000	64.9000	36.4000	36.4000	36.4000	102.600	102.600	102.600
.5	-21.5767	-20.1516	42.3478	- 58.7772	- 55.8866	- 30.0421	- 53.6414	-111.695	- 19.5154
1.0	-54.1625	-70.3632	-31.8132	- 96.0035	-114.403	- 70.5858	- 89.6547	- 70.0346	- 64.3959
1.5	-68.9698	-66.2033	-52.5367	-112.953	-110.415	- 94.1711	-126.020	-127.009	- 87.8033
2.0	-76.4928	-82.4401	-64.8773	-121.563	-127.679	-108.274	-114.329	-106.474	-101.493
2.5	-80.5090	-79.1129	-72.443	-126.147	-125.099	-116.930	-118.762	-125.965	-109.856
3.0	-82.7071	-84.8098	-77.1698	-128.671	-130.684	-122.338	-121.187	-118.308	-115.076
3.5	-83.9260	-83.3304	-80.1584	-130.064	-129.645	-125.756	-122.531	-124.975	-118.375
4.0	84.6067	-85.3372	-82.0623	-130.843	-131.508	-127.934	-123.282	-122.251	-120.475
6.0	-85.3915	-85.4783	-84.8906	-131.740	-131.814	-131.167	-124.148	-124.018	-123.595
8.0	-85.4703	-85.4807	-85.377	-131.830	-131.838	-131.723	-124.235	-124.218	-124.132
10.0	-85.4783	-85.4796	-85.461	-131.839	-131.840	-131.820	-124.243	-124.241	-124.225

	CNFRWD	CNFWBK	CNBACK
Execution Time	3 secs.	4 secs.	4 secs.
Time Step	.085	.5	.5

former requires accurate space simulation conditions often prevent "hardware" verification. Any uncertainty that is present merely compounds the correlation difficulties.

The important consideration in the verification of a mathematical model is not the duplication of the expected space conditions but rather the need for a describable chamber environment. Any uncertainty associated with the environmental facility must be kep small, otherwise, the temperature uncertainty interval becomes too broad. Verification of the "hardware" requires duplication of the space conditions unless, of course, it can be shown that less-than-ideal environmental conditions are adequate. A more detailed discussion on test philosophies is contained in Reference B-69.

B.3.2 Potential Sources of Uncertainties in the Chamber Environment

The potential sources of uncertainties in the environmental facility are numerous; the more sophisticated the system, the larger the number of potential sources of uncertainties. For example, a facility with earth emission and albedo radiation simulators in addition to the solar simulator is much more susceptible to environmental variances than a facility with only a solar simulator.

B.3.2.1 Solar Simulator

The types of solar simulator available on the market and presented being utilized in facilities are almost countless. At the beginning carbon-arc sources were most frequently employed but recently the trend has been away from carbon-arcs and toward xenon or mercury-xenon lamps.

The solar simulator characteristics are discussed below:

(1) Magnitude of Incident Flux Density - Aside from the capability of the solar simulator to maintain a specified intensity level, the uncertainty is directly related to the uncertainty of the data acquisition system, in particular, the accuracy of the radiometer and the recording equipment. This is discussed in paragraph 4. The variance of the intensity due to the instability of the source is directly related to the continuity of the intensity measurement during the test. In many facilities, the intensity is measured before and after a test and sometimes less frequently. Intensity changes that may

occur during a test are never known. Significant temporal instability is indicated by the results of Reference B-74 for a specific carbon-arc system. Xenon lamps have much better temporal instability. The temporal stability scans indicated a definite periodicity which is a function of the positive carbon advancement. The results indicated in intensity change of 10%.

(2) Uniformity of Incident Flux - Flux uniformity as used in the present discussion refers to the equality of the incident flux throughout the three dimensional region occupied by the test article. Again, the uncertainties of the spacewise intensity variation are directly dependent upon the quantity and the quality of the measurement system. Very often, spacewise flux intensity measurement are not made. If they are, the traverse is conducted at very infrequent intervals. Spacewise intensity variations of greater than 10% are not uncommon. The results of Reference B-74 indicate spatial intensity variations greater than 30% for a carbon-arc system.

(3) Spectral Energy Distribution - The spectral energy distribution in the test volume has been of much concern to users of solar simulators. Its importance is dependent not only upon the spectral characteristics of the energy irradiated surfaces but upon the test philosophy adopted for the test. For the interested reader, effect of solar simulation sources on the absorptance of several spacecraft coatings is tabulated in Reference B-62.

(4) Other Considerations - Other characteristics such as collimation and repeatability are either related to the uncertainty of the flux intensity or to the uncertainty associated with extraneous radiation. These parameters will not be discussed individually.

B.3.2.2 Albedo, Earth Emission and Lunar

No attempt will be made to assess the uncertainties of these secondary energy simulators. Several papers on this subject are listed in the reference section.[B-75 to B-78]

B.3.2.3 Chamber-Test Article Interactions

The thermal interactions between the test article and the chamber including the solar simulator can be quite complex with a resultant difficulty in defining the thermal environment to the test article. Many factors such as the geometry of the test article and its size, the optical system of the solar simulator, and the non-isothermal nature of the cold wall contribute to intricacies of the thermal interactions.

Needless to say, accurate assessment of the chamber environment must be specific and general guidelines must be used only in that context. General guidelines are reported in References B-63, B-65, B-66, and B-79 among others. It will suffice here to merely mention potential sources of uncertainties due to the test article-chamber thermal interactions. These include:

(1) extraneous solar reflection fron non-black walls;

(2) extraneous thermal radiation from uncooled mirrors, ports, and other "hot" spots;

(3) interaction between the test article and the simulator.

B.4 EXPERIMENTAL ERRORS - DATA ACQUISITION

Errors associated with data acquisition range from temperature perturbations produced by a sensor to the inaccuracies of the recording system. Clearly, an exhaustive treatment is beyond the intent of the present study and yet an in-depth examination of the data acquisition system to be employed for a thermal test is necessary to accurately assess the experimental errors. Since neither the in-depth study nor the particulars of the data acquisition system are available, the discussion here can only hope to indicate, in generalities, the magnitude of expected experimental errors.

B.4.1 Thermal Radiation Measurements

Parameters that describe the thermal radiation include: (1) spectral intensity distribution; (2) intensity; (3) collimation; and (4) uniformity of intensity. Radiometers are generally used to measure the intensity and the uniformity. Special instrumentation is required to measure the spectral irradiation and the collimation.

B.4.1.1 Intensity and Uniformity

Radiometers are of various types, many of which are of the flat surface variety. The accuracy of these radiometers are probably no better than $\pm 2\%$. The problems of precision radiometry are reported in References B-47, B-48, B-49. Measurement of solar simulator flux intensity is discussed in Reference B-80.

A TRW developed Model 43A radiometer which is presently being used in a number of environmental facilities in the country is accurate to

about ±2%. More recently and absolute cavity radiometer has been developed with an accuracy better than 2% (Reference B-81).

Eppley radiometers are commonly used for measuring total intensity from the solar simulator. The older Eppley pryoheliometers were subject to large uncertainties are indicated by results of References B-82 and B-83. Apparently, the new Eppley radiometer is more trouble-free.

In addition to the radiometer accuracy, the recording precision is estimated to be about ±1%. As a result, the total system error for measuring the flux intensity will be greater than ±3%.

B.4.1.2 Spectral Distribution and Collimation

No attempt will be made to assess the accuracy of these measurements. Reading material includes References B-74 and B-70; a collimation angle measurement device is described in B-74 and spectral measurements in B-74 and B-80. It should be recognized that the required accuracy of the spectral distribution and the collimation is highly dependent upon the spectral characteristics of the irradiated surfaces and upon the geometric characteristics of the test article.

B.4.2 Temperature Measurements

Many ways are available for measuring temperatures of solid surfaces, but thermocouples are by far the most commonly used. A number of factors must be considered to arrive at the system accuracy of the temperature measurements. Disruption of the measured surface by the presence of the thermocouple and the effect of lead losses are studies in themselves (References B-84 to B-87). Two other considerations are errors due to the quality of the wire and the reference junction. For premium grade wire (ISA specification) in the temperature range -75°F to 200°F, the error is ±3/4°F; for a regular grade, the error is ±1-1/2°F. An ice bath reference junction, well-designed and maintained, has an error about ±1°F. A cold junction compensator has an error that depends upon the ambient range. Typically the error is 1% of the range within the temperature bounds -65°F to 165°F.

The recording error depends upon the measured temperature, but typically the error is about ±1°F. Thus, the total system thermocouple error is about ±3°F.

B.5 UNCERTAINTIES ASSOCIATED WITH THE DEFINED SPACE ENVIRONMENTAL CONDITIONS

The primary source of thermal energy is the direct solar radiation with albedo and earth emission being secondary sources. For an orbit around the moon, lunar emission certainly becomes an important consideration. Information on the variances associated with the natural thermal environment in space contained in References B-63, B-88, and B-89 will be condensed and augmented by more recent information.

B.5.1 Solar Model

(1) Magnitude of Incident Flux Density: For earth orbiting and cis-lunar space vehicles, the solar flux ranges from (1.034)S at perihelion to (.966)S at aphelion. The value of the solar constant, S, which is the mean value, is the subject at the present of much discussion.[B-90, B-91] In the USA the results of Johnson is commonly used;[B-92] the solar constant is given as 442 Btu/hr ft^2 (2.0 cal/cm^2-min) with an uncertainty of $\pm$2%. In Europe, the evaluation of Nicolet is widely used;[B-93] the solar constant is considered to be 437 Btu/hr ft^2 (1.98 cal/cm^2-min) with an uncertainty of $\pm$5%. The experiment results of Reference B-94 and B-95 suggest that the Johnson solar constant is high by 2.5% and the Nicolet solar constant is high by 1.5%.

(2) Uniformity of Solar Flux: The solar energy incident upon surfaces in the vicinity of the earth arrives with a radius of curvature so large that the wave is assumed to be planar.

(3) Solar Flux Collimation: The terms collimation and decollimation are subject to variety of interpretations. A discussion of these terms is found in Reference B-96. The solar field angle in the vicinity of the earth is 32 minutes of arc.

(4) Spectral Energy Distribution: In the wave length of interest (for thermal analysis purposes), 0.2μ to 3.0μ, the spectral energy distribution is known relatively well, although there is some disagreement among investigators. In the USA, the spectral model of Johnson[B-92] is widely used and in Europe, the model of Nicolet[B-93] is popular. The two distributions are shown in Figure 1; note that large differences exist in the uv region. The results of Reference B-94 and B-95 suggest that the Johnson model below .6μ is about 6% high.

B.5.2 Earth Emission

A number of factors such as cloud coverage (including height), earth surface temperature, and moisture content of the atmosphere effect the earth emission. The magnitude of this energy is a stong function of

the latitude. In Reference B-87, a value in the range 75-78 Btu/hr ft^2 for a polar orbit is suggested. Measurements from OGO-II indicates a variation from 79 Btu/hr ft^2 to 124 Btu/hr ft^2 with a mean value averaged over a year for latitudes less than 30° of 101 Btu/hr ft^2.[B-97] Until more data become available, it appears that the commonly used value of 68 Btu/hr ft^2 is consistent with prevailing information. Perhaps an uncertainty interval of ±30% should be attached. Additional reading material includes References B-98 and B-99.

B.5.3 Albedo Radiation

Accurate analytical assessment of the albedo radiation is an extremely difficult undertaking because of the number of variables and the unknown factors involved. For a detailed discussion the reader is directed towards References B-100 and B-101.

For the planet earth, an average albedo of approximately 38% is commonly used although local variations are quite large, perhaps ranging from 10% to 90%.

Reference B-99 reported a variation of albedo from 0.10 to 0.62 between 33°N and 33°S latitudes on March 1965 with an average albedo of 0.29.

Lack of complete local albedo values coupled with analytical complexities in predicing reflected radiation dictages the use of average albedo value. The uncertainty in the use of the albedo radiation is increased further by the lack of accurate information on the spectral intensity distribution (References B-100, B-101, and B-102). Expediency tempered with engineering judgement dictates the use of a spectral intensity pattern that is the same as the direct solar radiation. An uncertainty interval of ±25% in the albedo appears to be a reasonable estimate.

B.5.4 Lunar Emission

The lunar emission from the moon to a spacecraft on its surface or in a lunar orbit is of extreme importance. The present state-of-knowledge about the emission characteristics of the moon is meager. Bi-directional reflectance behavior of lunar-like materials indicates that the thermal emission has a directional flavor.[B-103] A recent lunar infrared scan data reveal directional effects.[B-104] Thermal engineering data from

Surveyor I indicate a non-Lambertion surface thermal emission effects.[B-105] At the present time, for thermal design analysis it is a common practice to assume that lunar emission follows a cosine distribution.

Perhaps a reasonable upper limit at the subsolar point is 433 Btu/hr ft^2. No attempt will be made to assess the uncertainty interval.

B.6 REFERENCES

B-1 Clausen, O. W. and Ishimoto, T., "Thermal Test of a Model Space Vehicle, Part I., Description and Thermal Analysis of Model", AFFDL-TR-67-42, Part I, June 1967.

B-2 Ishimoto, T. and Bevans, J. T., "Temperature Variance in Spacecraft Thermal Analysis", AIAA Paper No. 68-62, AIAA 6th Aerospace Sciences Meeting, January 22-24, 1968, New York.

B-3 Kline, S. J. and McClintock, F. A., "Describing Uncertainties in Single-Sample Experiments", Mechanical Engineering, Vol. 75, 1953, pp 3-8.

B-4 Gill, W. L., "MSC Thermophysical Property Library", NASA/MSC (To be published in 1969).

B-5 Edwards, D. K., "Uncertainties in Spacecraft Thermal Radiation Properties", TRW IOC 65-9723.3-255.

B-6 Grammer, J. R., "Emissivity Coatings for Low-Temperature Space Radiators", Quarterly Progress Report No. 5, 30 September 1966, NASA-CR-72130.

B-7 Nelson, K. E. and Bevans, J. T., "Errors of the Calorimetric Method of Total Emittance Measurement", Measurement of Thermal Radiation Properties of Solids, NASA SP-31, 1963 (Symposium held at Dayton, Ohio, September 4, 6, 7, 1962).

B-8 Luedke, E. E., "Emittance of '3M' Corporation 401-C10 'Velvet Coating'," TRW IOC 66-3331.11-60, 28 June 1966.

B-9 Atkins, H., "Bibliography on Thermal Metallic Contact Conductance", NASA TM-X-53227, April 1965, Marshall Space Flight Center.

B-10 Fried, E., "Thermal Joint Conductance in a Vacuum", ASME 63-AHGT-18.

B-11 Fried, E. and Kelley, M. J., "Thermal Conductance of Metallic Contacts in a Vacuum", AIAA Paper No. 65-661, AIAA Thermophysics Specialists Conference, September 1965, Monterey, California.

B-12 Clausing, A. M. and Chao, B. T., "Thermal Contact Resistance in a Vacuum Environment", ME-TN-242-1, August 1963, Engineering Experiment Station, University of Illinois.

B-13 Henry, J. J. and Fenech, H., "The Use of Analog Computers for Determining Surface Parameters Required for Prediction of Thermal Contact Conductance", ASME Paper No. 63-WA-104.

B-14 Stubstad, W. R., "Measurements of Thermal Contact Conductance in Vacuum", ASME Paper No. 63-WA-150.

B-15 Barzelay, M. E., "Range of Interface Thermal Conductance for Aircraft Joints", NASA TN D-426, May 1960.

B-16 Lindh, K. G., Lieb, B. A., Knuth, E. L., Ishimoto, T., and Kaysen, H. M., "Studies in Heat Transfer in Aircraft Structure Joints", Report No. 57-50, May 1957, UCLA.

B-17 Lindh, K. G., "Measurement of Thermal Contact Resistance", UCLA, Engineering Report No. C14-53, July 1953.

B-18 Aron, W. K. and Colombo, G., "Controlling Factors of Thermal Conductance Across Bolted Joints in a Vacuum", ASME Paper 63-W-196.

B-19 Bevans, J. T., and Ishimoto, T., Loya, B. R., and Luedke, E. E., "Prediction of Space Vehicle Thermal Characteristics", AFFDL-TR-65-139, October 1965.

B-20 Rolsma, B., "Evaluation of Contact Conductance at Critical Joints in the Apollo Spacecraft, Volume I", GE Document No. 66SD4487, 16 September 1966.

B-21 Rolsma, B., "Evaluation of Contact Conductance of Critical Joints in the Apollo Spacecraft, Volume II - DATA", GE Document No. 66SD4487, 16 December 1966.

B-22 Deem, H. W. and Lucks, C. F., "An Improved All-Metal Bunsen-Type Calorimeter", Conference of the Instrument Society of America, Battelle Memorial Institute, Columbus, Ohio, September 1958.

B-23 Newnam, B. E., "Enthalpy and Specific Heat of Magnesium Alloy, LA 141A-T7, TRW IOC 67-3346.11w-6, February 1967.

B-24 ASTM Standards 1961, American Society for Testing and Materials, Standard Method of Test for Thermal Conductivity of Materials by Means of the Guarded Hot Plate, pp. 953-961.

B-25 Wechsler, A. E. and Glaser, P. E., "Investigation of the Thermal Properties of High-Temperature Insulation Materials", ASD-TDR-63-574, July 1963, p 29.

B-26 Haroldsen, O. O., Pence, W. M., and Elizalde, J. C., "Cryogenic Insulating Development Program, Phase II Report", 9990-7306-T0000, July 1966, TRW Systems.

B-27 Elizalde, J. C., "Measurement and Effect of Residual Pressure in Foil Insulation", TRW IOC 66-3331.4-13, October 7, 1966.

B-28 Bacha, C. P., Honea, F. I., and Watanebe, D., "Temperature Control Systems for Space Vehicles", ASD-TDR-62-493, Part I, May 1963, p. 65.

B-29 Loya, B. R., "Effective Emissivity and Absorptivities of a Full Open Specular Louver System Irradiated by Diffuse Solar and Infrared Energy", TRW IOC 63-9723.1-50, 27 May 1963.

B-30 Plamondon, J. A., "Analysis of Movable Louvers for Temperature Control", Technical Report No. 32-555, January 1, 1964, JPL.

B-31 Moon, P. and Spencer, D. E., "Optical Transmittance of Louver Systems", Journal of the Franklin Institute, Vol. 273, No. 1, January 1962.

B-32 Wiebelt, J. A., Parmer, J. F., and Kneissl, G. J., "Spacecraft Temperature Control by Thermostatic Fins-Analysis", NASA CR-155, January 1965.

B-33 Greenfield, M. L., "OGO Louver Model Test Results", TRW IOC 64-9723.11-3, 11 February 1964.

B-34 Haroldsen, O. O., "Emissivity of 169 Louvers", TRW IOC 67-3346.9j-4, 6 March 1967.

B-35 Karplus, W. G., Analog Simulation, Solution of Field Problems, McGraw-Hill Book Co., 1958.

B-36 Tribus, M., "The Use of Analogs and Analog Computers in Heat Transfer", Publication No. 100, May 1958, Oklahoma State University.

B-37 Barakat, H. Z. and Clark, J. A., "On the Solution of the Diffusion Equations by Numerical Methods", J. of Heat Transfer, November 1966.

B-38 Anderson, J. T. and Botze, J. M., "Spatial Truncation Error Analysis", ASME 62-HT-27.

B-39 Anderson, J. T., "A Review of Digital-Computer Heat Transfer Programs," ASME 65-WA/HT-48.

B-40 Friedman, H. E., "The Truncation Error in a Semi-Discrete Analog of the Heat Equation", Journal of Mathematics and Physics, Vol. 35, No. 3, October 1956.

B-41 Friedman, H. E., "Quasilinear Heat Flow", PhD. Dissertation, October 1956, Chapter 5.

B-42 Kardas, A., "Errors in a Finite Difference Solution of the Heat Flow Equation", J. of Heat Transfer, November 1964.

B-43 Bobco, R. P., "Gray Surface Radiation Between a Differential Area and an Infinite Plane: Nonuniform Local Heat Flux", ARS Journal, August 1962.

B-44 Sparrow, E. M., "Heat Radiation Between Simply-Arranged Surfaces Having Different Temperatures and Emissivities", A.I.Ch.E. Journal, March 1962.

B-45 Sparrow, E. M. and Cess, R. D., "Radiation Heat Transfer", Wadsworth Publishing Co., 1966.

B-46 Nolte, L. J., "Radiation Interchange Factors", First Quarterly Report, 7 June through 1 September 1968, Hughes Aircraft Co., SSO 80339M, NASA Contract NAS 9-7980.

B-47 Drummond, A. J., "Precision Radiometry and Its Extension to Aspects of Space Physics", Course: Fundamental Radiometry for Experimental Scientists, Eppley Laboratories, Inc. October 13, 1965.

B-48 Bedford, R. E., "Blackbodies as Absolute Radiation Standards", Course: Fundamental Radiometry for Experimental Scientists, Eppley Laboratories, Inc., October 13, 1965.

B-49 Stair, R., "Sources as Radiometric Standards", Course: Fundamental Radiometry for Experimental Scientists, Eppley Laboratories, Inc., October 13, 1965.

B-50 Bussell, R., "Properties of a Variable Structure, Computer System in the Solution of Parabolic Partial Differential Equations", Report No. 62-46, September 1962, UCLA.

B-51 Strong, P. F. and Emslie, A. G., "The Method of Zones for the Calculation of Temperature Distribution", ASME 65-WA/HT-47.

B-52 Barker, R. S., "Method of Sampled Perturbation Applied to Transient Heat Flow Problems", 63-AHGT-92.

B-53 Jones, B. P., "A Study of Several Numerical Methods for Solving a Particular System of Ordinary Differential Equations", TM X-53121, NASA/MSFC (no date given).

B-54 Edwards, A. L., "Trump: A Computer Program for Transient and Steady State Temperature Distributions in Multidimensional Systems", UCRL-14754, Rev. 1, TID-4500, UC-32, May 1, 1968, Lawrence Radiation Laboratory.

B-55 Engvall, J. L., "An Engineering Expose of Numerical Integration of Ordinary Differential Equations", NASA TN D-3696, November 1966.

B-56 Douglas, Jr., J., "Round-Off Error in the Numerical Solution of the Heat Equation", J. Assoc. Computing Machinery 6:48-58, January 1959.

B-57 Lewis, D. R., Gaski, J. D., and Thompson, L. R., "Chrysler Improved Numerical Differencing Analyzer for 3rd Generation Computers, CINDA 3G", TN-AP-67-287, October 20, 1967.

B-58 Dusinberre, G. M., Numerical Analysis of Heat Flow, McGraw-Hill Book Co., Inc., New York, 1949.

B-59 Levy, H. and Baggott, Numerical Solutions of Differential Equations, Dover Publications, Inc., 1950.

B-60 Solar Radiation Simulation - State of the Art Survey, IES, January 1965.

B-61 Hollingsworth, R. T., "A Survey of Large Space Chambers", NASA TN D-1673, April 1963.

B-62 Bobco, R. P. and Ishimoto, T., "A Suggested Solar-Simulation Standard for Thermal Testing", 64-WA/HT-17.

B-63 Bobco, R. P., and Ishimoto, T., "Temperature Errors in Simple Systems Caused by Deviations from Ideal Space-Solar Simulation", International Symposium on Solar Radiation Simulation Proceedings, January 1965.

B-64 Gooze, E. G., and Brooks, J. J., "Analysis of Thermal Test Errors Resulting from Imperfectly Collimated Solar Simulation Beams; Phase I", JPL 66-196, 21 March 1966.

B-65 Kelley, L. R., "Thermal Simulation Errors with Emphasis on the Collimated Beam Test Method", 99900-6088-R000, 18 December 1966, TRW Systems.

B-66 Kelley, L. R., "Thermal Simulation Errors Associated with the Zone-Controlled Heating Heat Method", 99900-6097-R000, 18 December 1966, TRW Systems.

B-67 MacFarlane, G., "Solar Simulation: Requirements for Large Aerospace Environmental Chambers", AEDC-TR-65-164, August 1965.

B-68 Anderson, J. W., LaBlanc, E. A., and Cohan, H., "Experimental and Analytical Assessment of Space Thermal and Vacuum Environmental Simulation Requirements", Lockheed Missiles and Space Co., 1965.

B-69 Jaatinen, W. A., Rothacker, D. L., Bull, R. H., Dachs, M. A., and Shenker, M. R., "Solar Radiation Simulation Studies, Part I", AEDC-TDR-63-90, May 1963.

B-70 Jaatinen, W. A. Rothacker, D. L., Fitz, C. D., Bull, R. H., Dachs, M. A., and Shenker, M., "Solar Radiation Simulation Studies, Part 2, AEDC-TDR-63-91, May 1963.

B-71 Fischel, H., and Hughes, R. R., "A Study of Optical System Concepts and High Intensity Light Sources for Solar Radiation Simulation", AEDC-TR-65-117, May 1965.

B-72 "Solar Simulation Research", X-633-65-332, August 1965. Goddard Space Flight Center.

B-73 Ishimoto, T., and Bevans, J. T., "Test Philosophies and Methods", TRW Report No. 99900-6103-R000, 31 December 1966.

B-74 Alexander, S., "The Spectral Distribution and Uniformity of the Carbon Arc", Report X-713-66-79, March 1966, NASA Goddard.

B-75 Fitz, C. D. and Mayer, E. A., "Evaluation of Methods for Simulating the Earth's Albedo and Planet Radiation", AEDC-TDR-63-217, October 1963.

B-76 Camack, W. G., "Method for Simulating Planetary Radiation", November 1964, Philco.

B-77 International Symposijm on Solar Radiation Simulation Proceedings, IES and ASTM, January 18-20, 1965, Los Angeles, California.

(a) "Albedo and Planetary Radiation Simulation", W. A. Henry and F. U. Williams

(b) "Effectiveness of Albedo Earth Radiation Simulation", C. D. Fitz

(c) "The Design of an Albedo and Planetary Emission Simulator", L. B. Kelley

B-78 Space Simulation Conference, AIAA/IES/ASTM, September 7-9, 1966, Houston, Texas.

(a) "Adequacy Considerations for Lunar Environmental Simulation", J. C. Simons, Jr.

(b) "Thermal Simulation of the Lunar Surface", R. D. Wood and J. N. Wilkinson

B-79 Wainwright, J. B., Kelley, L. R., Wallace, D. A., and Keesee, T. H., "Modeling Criteria and Testing Techniques for the Simulation of Space Environmenta", AFFDL-TR-64-164, October 1964.

B-80 Drummond, A. J. and Hickey, J. R., "Measurement of the Total Flux and Its Spectral Components in Solar Simulation Systems with Special Reference to the Extraterrestrial Radiation", Solar Energy Vol. 10, No. 4, 1966.

B-81 Kendall, J. M. and Plamondon, J. A., "An Absolute Cavity Radiometer for the Measurement of Thermal Radiometer", AIAA/IES/ASTM Space Simulation Conference, September 1966.

B-82 Nelson, K. E., "Effect of Orientation on Eppley Pyrheliometer Calibration", TRW IOC 9723.3-19, 12 February 1963.

B-83 Johnson, A. W., "Comparison of Solar Simulator Intensities as Measured with an Eppley Phrheliometer and a Black Disc Radiometer Target Board", TRW IOC 9723.4-36, 1 July 1963.

B-84 Beck, J. W. and Hurwicz, H., "Effects of Thermocouple Cavity on Heat Sink Temperature", Journal of Heat Transfer, Trans. ASME, Series C, Vol. 82, 1960, pp 127-136.

B-85 Pfahl, R. C., Jr. and Dropkin, D., "Thermocouple Temperature Perturbations in Low Conductivity Materials", ASME Paper No. 66-WA/HT-21.

B-86 Henning, C. D. and Parker, "Transient Response of an Intrinsic Thermocouple", ASME Paper No. 66-WA/HT-21.

B-87 Kelley, L. R., "Analysis of Local Temperature Error Induced by Lead-Wire Conduction", TRW IOC 67-3346.11n-01, 20 February 1967.

B-88 Camack, W. G. and Edwards, D. K., "Effect of Surface-Thermal-Radiation Characteristics on the Temperature Control Problem in Satellites", First Symposium, Surface Effects on Spacecraft Materials, Edited by F. J. Clauss, John Wiley and Sons, N. Y., 1960.

B-89 Herold, L. M., "Design Values of Solar Radiation, Albedo and Earth Radiation", TRW IOC 66-3331.10k-2, 14 June 1966.

B-90 Thekaekara, M. P., "The Solar Constant and Spectral Distribution of Solar Radiant Flux", Solar Energy, Vol. 9, No. 1, pp 7-20 (1965).

B-91 Drummond, A. J., "The Extraterrestrial Solar Spectrum", International Symposium on Solar Radiation Simulation Proceedings", IES and ASTM, January 18-20, 1965.

B-92 Johnson, F. S., "The Solar Constant", Journal of Meteorology, Vol. 11, December 1954.

B-93 Nicolet, M., "Sur la de'termination du flux energetique du rayonnement extra terrestre du soleil", Archiv fur Meterologie, Geophysik, und Bioklimatalogie, Series B, Geophysik und Bioklimatologie, Vol. 3, 1951.

B-94 Drummond, A. J., Hickey, J. R., Scholes, W. J., and Laue, E. G., "Multichannel Radiometer Measurement of Solar Irradiance", AIAA Paper No. 67-147, AIAA 5th Aerospace Sciences Meeting.

B-95 Laue, E. G., and Drummond, A. J., "Solar Constant: First Direct Measurements", Science, 30 August 1968, Vol. 161, pp 888-891.

B-96 Fischel, H., "Some Thermodynamic Principles of Radiative Transfer in Optical Systems", Fifth Annual Symposium on Space Environment Simulation, AEDC, Tullahoma, Tenn., May 1964.

B-97 Pearson, Jr., B. D., and Neel, C. B., "Albedo and Earth Radiation Measurements from OSO-II", AIAA 67-330, AIAA Thermophysics Specialist Conference, April 17-20, 1967.

B-98 Allison, L. J., "An Analysis of Tiros II Radiation Data Recorded Over New Zealand at Night", NASA TND-1910, March 1964.

B-99 Milliard, J. P. and Neel, C. B., "Albedo and Earth Radiation Deduced from Emissivity Sensors on the First Orbiting Solar Observatory," AIAA Paper No. 64-317, 1st AIAA Annual Meeting, June 29 - July 2, 1964.

B-100 Snoddy, W. C., "Irradiation Above the Atmosphere Due to Rayleigh Scattering and Diffuse Terrestrial Reflections", AIAA Paper No. 65-666, AIAA Thermophysics Specialists Conference, September 13-15, 1965.

B-101 Whitehill, L. P., "Survey on Earth Albedo", TN 1966-53, 11 October 1966, MIT.

B-102 Camach, W. G., "Albedo and Earth Radiation", IES: 1962 Proceedings, pp 391-396 (1962).

B-103 Willingham, D., "The Lunar Reflectivity Model for Ranger Block III Analysis", JPL Technical Report No. 32-664, 2 November 1964.

B-104 Six, N. F., Montgomery, C. G., Saari, J. M., and Shorthill, R. W., "Directional Characteristics of the Lunar Thermal Emission", AIAA 67-291, AIAA Thermophysics Specialist Conference, April 17-20, 1967 (from abstract).

B-105 Lucas, J. W., Conel, J. E., Hagemeyer, W. A., Caripay, R. R., and Saari, J. M., "Lunar Surface Thermal Characteristics from Surveyor I Data", AIAA 67-292, AIAA Thermophysics Specialist Conference, April 17-20, 1967 (from abstract).

APPENDIX C. OBSERVABILITY OF THERMAL NETWORKS

In general, the ability to carry out meaningful thermal network correction is dependent on whether or not the network variables which are observed or measured are functionally dependent (in an appropriate sense to be defined below) on the parameters and/or initial temperatures whose values are to be corrected. Because of the very real possibility that the appropriate functional dependence will not exist in situations of engineering significance, it is important that some attention be given to the problem of ascertaining its existence or lack of existence. In the terminology of modern systems theory, such a problem falls under the general heading observability in nonlinear systems. In this section, the results of a brief investigation into the observability of thermal networks is presented. These results are rather incomplete and fragmentary, and they are presented at this time only to indicate the nature and significance of the general problem of observability of nonlinear systems and its relevance to thermal network correction. Clearly, much more effort could, and probably should, be applied in this area.

Consider a general thermal network which can be characterized mathematically by a vector differential equation of the form

$$\dot{T} = f(t,T,p) \tag{C-1}$$

where the components of the vector T denote the temperatures at the nodes of the network and the components of the parameter vector p denote the values of the thermal conductances, thermal radiation exchange coefficients, heat capacities, etc. for the network.* In the general case certain of the parameters and initial temperatures can be taken as being known (or hard) while the others are to be corrected based on a set of temperature measurements or observations made at selected nodes (i.e., are soft). Let p^1 be the vector whose components are the soft parameters. Similarly, let T_o^1 be the vector whose components are the hard initial temperatures while T_o^2 is a vector whose components are the soft initial temperatures. Finally, let $T^1(t)$ be the vector whose components are the temperatures of the nodes which are measured and let $T^2(t)$ denote a vector

* It is tacitly assumed here tha all inputs to the network are known functions of time.

whose components are the temperatures of the nodes which are not measured. It may or may not be true that $T^1(t_o) = T^1_o$ and $T^2(t_o) = T^2_o$. Then Equation (C-1) can be written in an expanded form as follows:

$$\dot{T}^1 = f^1(t,T^1,T^2,p^1,p^2) \tag{C-2}$$

$$\dot{T} = f^2(t,T^1,T^2,p^1,p^2) \tag{C-3}$$

If certain, rather nonrestrictive, conditions are imposed on the functions $f^1(\ )$ and $f^2(\ )$, unique solutions of Equations (C-2) and (C-3) exist for any given set of values for T^1_o, T^2_o, p^1, and p^2. Moreover, if T^1_o and p^1 are assumed fixed and T^2_o and p^2 are assumed variable, then the family of solutions of Equations (C-2) and (C-3) which are obtained by varying T^2_o and p^2 defines $T^1(t)$ and $T^2(t)$ as functions of T^2_o and p^2 for all t. Let the functions so defined be denoted by the expressions

$$T^1(t) = g^1(t,T^2_o,p^2) \tag{C-4}$$

$$T^2(t) = g^2(t,T^2_o,p^2) \tag{C-5}$$

Now suppose a set of observations of $T^1(t)$ are made at times t_o, t_1,..., t_N and let $\{T^o(t_k);\ k=o,...,N\}$ denote this set of observations. Then it is reasonable to define the observability of T^2_o and p^2 as follows.

Definition 1: T^2_o and p^2 will be said to be simultaneously observable relative to the set $\{T^1(t_k);\ k=o...,N\}$ if and only if the (nonlinear) algebraic equations

$$\begin{aligned} T^1(t_o) &= g^1(t_o,T^2_o,p^2) \\ T^1(t_1) &= g^1(t_1,T^2_o,p^2) \\ &\cdot \\ &\cdot \\ &\cdot \\ T^1(t_N) &= g^1(t_N,T^2_o,p^2) \end{aligned} \tag{C-6}$$

have a unique solution for T^2_o and p^2.

Definition 2: T_o^2 and p^2 will be said to be simultaneously observable at time $\tau \geq t_o$ if and only if there exists some N and some set $\{t_k; k=o,\ldots,N\}$ with $t_o \leq t_k \leq \tau$ such that Equations (C-6) have a unique solution. Otherwise they will be said to be unobservable. If T_p^2 and p^2 are unobservable at time τ for all $\tau \geq t_o$, then they are simply said to be unobservable.

An obvious consequence of the above definitions is that if T_o^2 and p^2 are unobservable at time τ then it is impossible to determine the true values of T_o^2 and p^2 for the network from observations of $T^1(t)$ made on the interval $[t_o,\tau]$.

The above is actually a special case of a more general question of observability in nonlinear systems. This more general case can be formulated analytically as follows. Let x(t) denote a n-vector function of time which satisfies the differential equation

$$\dot{x} = f(t,x,p) \tag{C-7}$$

where p is a set of (constant) parameters. Let y(t) denote an m-vector function of time whose components correspond a nonlinear measurements of x(t), i.e.,

$$y(t) = h(x(t), t) \tag{C-8}$$

By the same argument used in the special case above the family of solutions of Equation (C-7) obtained by varying $x(t_o)$ and p define x(t) as an n-vector function of t, $x(t_o)$, and p which will be denoted by $g(t,x(t_o),p)$; i.e.,

$$x(t) = g(t,x(t_o),p) \tag{C-9}$$

Substitutions of Equation (C-9) and Equation (C-8), gives

$$y(t) = h(g(t,x(t_o),p),t) \tag{C-10}$$

Definition 3: The nonlinear system described by Equations (C-7) and (C-8) will be said to be state observable at p relative to the set of observations $\{y(t_k); k=o,\ldots,N\}$ if and only if the equations

$$\left.\begin{aligned} y(t_o) &= h(g(t_o,x(t_o),p),t_o) \\ y(t_1) &= h(g(t_1,x(t_o),p),t_1) \\ &\vdots \\ y(t_N) &= h(g(t_N,x(t_o),p),t_N) \end{aligned}\right\} \qquad \text{(C-11)}$$

have a unique solution for $x(t_o)$.

Similarly

Definition 4: The nonlinear system described by Equation (C-7) and (C-8) will be said to be state observable at time $\tau \geq t_o$ at p if and only if there exists some N and some set $\{t_k,\ k=o,\ \ldots,N,\ t_o \leq t_k \leq \tau\}$ such that Equation (C-11) have a unique solution for $x(t_o)$.

Analogous to state observability, parameter observability can be defined as follows:

Definition 5: The nonlinear system described by Equation C-7) and (C-8) will be said to be parameter observable for initial condition $x(t_o)$ at time $\tau \geq t_o$ if and only if there exists some N and some set $\{t_k;\ k=o,\ldots,N, t_o \leq t_k \leq \tau\}$ such that Equation (C-11) have a unique solution for p. If this unique solution is independent of $x(t_o)$ the system will simply be said to be parameter observable.

Definition 6: The nonlinear system described by Equation (C-7) and (C-8) will be said to be simultaneously state and parameter observable for a set of observations $\{y(t_k);\ k=o,\ldots,N\}$ if and only if Equations (C-11) have a unique solution for $x(t_o)$ and p.

Several comments relative to Definitions 3-6 are in order before returning to the special case of observability in thermal networks. First, it can be shown rather easily that Definitions 3 and 4 are equivalent to Kalman's definition of (state) observability for linear systems * when Equations (C-7) and (C-8) are linear. However, they appear to

* Kalman, R.E., "New Methods and Results in Linear Prediction and Filtering Theory, RIAS TR 61-1, Presented at the Symposium on Engineering Application of Random Function Theory and Probability, Purdue University, November 1960.

offer some advantage even in this case because the meaning of observability is more directly interpretable in engineering terms. Second Kalman's definition of observability for linear systems is not directly applicable to parameter observability in most cases of engineering interest. This is because, even though Equations (C-7) and (C-8) may be linear in x(t), the parameter enters the problem in such a way that $y(t_k)$ is a nonlinear function of p. This is illustrated by the simple scaler linear case where

$$\dot{x}(t) = px(t) \text{ and } y(t) = x(t) \tag{C-12}$$

for which it can be shown that

$$y(t_k) = x(t_k) = x(t_o)\, e^{p(t_k - t_o)} \tag{C-13}$$

In this case, it is seen that $y(t_k)$ is an exponential function of p. On the other hand, the usefulness of Kalman's definition hinged on the fact that the $y(t_k)$ were linear functions the quantities to be determined or estimated; i.e., linear functions of $x(t_o)$ in his case. Finally, Definitions 3 and 4 and especially Definitions 5 and 6 represent bona fide generalizations of Kalman's definitions in a form which is: (1) directly applicable to nonlinear systems with or without unknown or poorly specified parameters and (2) immediately interpretable in terms of the problem determining or defining estimates of the initial conditions and/or parameters of such systems. The implications of these definitions will become apparent from the discussion to follow.

There is a close relationship between observability as defined above and the ability to correct initial estimates or guesses of $x(t_o)$ and p based on observations $y(t_k)$. This relationship can be seen as follows. Let $\underline{x}(t_o)$ and $\underline{p}$ denote initial estimates of $x(t_o)$ and p. Then define correctability in the following manner.

Definition 7: The estimates of $\underline{x}(t_o)$ and $\underline{p}$ will be said to be correctable given a set of observation $\{y(t_k);\ k=o,\ldots,N\}$ of the system if and only if it is possible to find corrections $\delta x(t_o)$ and δp (which are functions of the $y(t_k)$) such that $\underline{x}(t_o) + \delta x(t_o)$ and $\underline{p} + \delta p$ are equal to initial condition and parameter values which produced the $y(t_k)$; i.e., such that $x(t_o) = \underline{x}(t_o) + \delta x(t_o)$ and $p = \underline{p} + \delta p$.

The following conclusions can be established based on the above definitions.

Conclusion 1: In order for estimates $\underline{x}(t_o)$ and $\underline{p}$ to be correctable given a set of observations $\{y(t_k);\ k=o,\ldots,N\}$, it is necessary that the system be simultaneously state and parameter observable given this set of observations.

Conclusion 2: Let the system be simultaneously state and parameter observable given the set of observations $\{y(t_k);\ k=o,\ldots,N\}$ and let the mN - vector be defined by the relationship

$$y = \begin{bmatrix} y(t_o) \\ - - - \\ \cdot \\ \cdot \\ \cdot \\ - - - \\ y(t_N) \end{bmatrix}$$

Then, if the matrix $[\frac{\partial Y}{\partial x(t_o)} \vdots \frac{\partial Y}{\partial p}]$ has rank equal to the number of components of $x(t_o)$ plus the number of components of p, the estimates $\underline{x}(t_o)$ and $\underline{p}$ are correctable for all $\underline{x}(t_o)$ and $\underline{p}$ in some neighborhood of $x(t_o)$ and p.

Conclusion 1 follows from the observation that the existence of $\delta x(t_o)$ and δp, with the prescribed property is equivalent to the existence of unique solutions of Equations (C-11). Conclusion 2 follows from Conclusion 1 and use of the implicit function theorem.

As an example of a thermal network which is not correctable consider the linear five node case described by the following equations

$$\dot{T}^1 \left\{ \begin{array}{l} \dot{T}_1 = a_{12}(T_2-T_1) + a_{13}(T_3-T_1) + a_{14}(T_4-T_1) + a_{15}(T_5-T_1) \\ \quad \cdot \\ \quad \cdot \\ \quad \cdot \\ \quad \cdot \\ \quad \cdot \\ \quad \cdot \\ \dot{T}_3 = a_{31}(T_1-T_3) + a_{32}(T_2-T_3) + a_{34}(T_4-T_3) + a_{35}(T_5-T_3) \end{array} \right.$$

$$\dot{T}^2 \quad \left\{ \begin{array}{l} \dot{T}_4 = a_{41}(T_1-T_4) + \ldots\ldots + a_{45}(T_5-T_4) \\ \quad \vdots \\ \dot{T}_5 = a_{51}(T_1-T_5) + \ldots\ldots + a_{54}(T_4-T_5) \end{array} \right.$$

where all of the a_{ij} are all equal and where the a_{ij} are taken to be the parameters. Direct observation of the equations indicates that $\dot{T}_1$ through $\dot{T}_3$ are dependent only on $T_4 + T_5$ and similarly $\dot{T}_4$ and $\dot{T}_5$ are dependent only on $T_1 + T_2 + T_3$. Hence, if T_4 and T_5 were initialized at $T + \delta$ and $T - \delta$, respectively, then $T_4(t) + T_5(t)$ would be functions of $T_4(t_o)$ and $T_5(t_o)$ only through their sum $T_4(t_o) + T_5(t_o)$. But, this means that the system is not simultaneously state and parameter observable for any set of observations of $T^1(t)$ because $T_4(t_o) = T + \delta$ and $T_5(t_o) = T - \delta$ would satisfy Equations (C-6) for all δ. That is, Equations (C-6) would not have a unique solutions for $T^2(t_o)$ and p and hence, by Conclusion 1 the values of $T_4(t_o)$ and $T_5(t_o)$ would not be correctable.

APPENDIX D. ERROR EFFECTS: TEMPERATURES & MATRIX INVERSION

D.1 EFFECTS OF SMALL TEMPERATURE MEASUREMENT ERRORS

The effects of small temperature measurement errors for the deterministic method presented in paragraph 3.1.1 can be computed as follows:

Let the set of equations be expressed in the general matrix form,

$$[M]\ \{x\} = 0 \tag{D-1}$$

where, [M] is the coefficient matrix
{x} is the parameter vector

If we let $[M_o]$ and $\{x_o\}$ be the true coefficeints matrix and parameter vector, respectively, and if we let $M = [M_o] + [\Delta M]$ be the coefficient matrix obtained by using noisy temperature measurements and $\{x\} = \{x_o\} + \{\Delta x\}$ be the resulting {x} which is computed using the noisy coefficient matrix [M], then

$$(M_o + \Delta M)(x_o + \Delta x) = 0 \tag{D-2}$$

Matrix algebra, use of equation (D-1) and the definition of [M] yields:

$$[M]\{x_o\} + [M]\{x\} = 0 \tag{D-3}$$

Or,

$$\{\Delta x_o\} = [M]^{-1}[\Delta M]\ \{x_o\} \approx [M_o]^{-1}[\Delta M]\{x_o\} \tag{D-4}$$

Equation (D-4) indicates that the error in the parameter x is approximately linear with error in the coefficient matrix M.

D.2 MATRIX INVERSION

The method of least squares discussed numerical round-off difficulties (paragraph 4.1.1). In particular the manner in which the coefficient matrix was inverted yielded surprisingly different results. The three matrix inversion subroutines may be described as follows:

All three subroutines, SØLVIT, INVRSE, and GJR use the same basic principle--i.e., reduction of the original matrix to the identity matrix by elementary row (or column) operations; the corresponding elementary operations on the identity matrix produce the inverse matrix*. Symbolically

$$[A,I] \rightarrow [I,A^{-1}]$$

The elementary operations discussed above are either: (1) division of a row by the diagonal element or (2) addition of a linear combinational of one row to another row. The matrices may be inverted in their own space by shifting each row left by one column as it is being reduced. Row interchange means that the largest element in a particular column of the unreduced matrix is used as the diagonal element. In row and column interchange the largest element of the entire unreduced matrix is used as the diagonal element. Both INVRSE and GJR yield the inverse explicitly whereas the subroutine SØLVIT augments the coefficient matrix [A] with the forcing function {B}. The process of reducing the matrix [A] to the identity matrix with corresponding operations on B produces the solution vector [x] in [B].

$$[A;B] \rightarrow [I;x]$$

---

* Froberg, C. E., Introduction of Numerical Analysis, 1965 Addison-Wesley

Kelly, L. G., Handbook of Numerical Methods and Application. 1967, Addison-Wesley.